# Mathematics Tools and its Efficiency on the Building's Environmental Performance

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## Abstract

Recently, mathematics has a wide influence on architectural design and performance. It can effectively help in solving design problems and promoting geometrical shapes efficiency. Consequently, the environmental performance has also widely developed with the involving of advanced mathematical branches and computational software. The main aim of the paper is to study the importance of using mathematics in raising the building's environmental performance. Therefore, the research methodology starts by clarifying the mathematics' branches and their application in architecture, then carries out an analysis for different case studies that used mathematics effectively in developing environmental performance. And lastly, apply one of the mathematical optimization techniques on a virtual prototype to illustrate how mathematics can effectively promote the building's environmental efficiency.

**Keywords** Mathematics, Geometry, Parametric, Algorithm, Topology, Environmental Performance.

## 1. Introduction

From ages, Architecture and mathematics had common basics and concepts. In history, architects were mathematicians and some mathematicians were architects too. Thus, architects had used mathematics by means of a tool in the developing of architecture projects, for example, the mathematical readings of Pythagoras that were used in building proportions [1]. Accordingly, that continued all overages till now in contemporary architecture where mathematics became a central core in the architecture from the initial stage of form-finding, shaping and generating complex forms to the fabrication process, via opening new horizons in architecture field [2].

In architectural design, *Architecture Performance* was a generic term that implies the ability to perform [3]. So, its evaluation criteria were multidisciplinary, thus some architects sorted architecture performance into environmental, structural, and social performance [4]. *Environmental performance*; which was selected in this research, that related to the built environment's quality, whether indoor or outdoor, *Structural performance;* that cares directly about occupants' safety in the buildings. While, *Aesthetic performance*; affected by form, material selection, and, color [3].

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### 1.1 Research Problem

Mathematics has revealed a wide range of methods and techniques with high potential in promoting environmental performance efficiency. Therefore, the non-awareness of these techniques can make architects miss several chances in optimizing the building's environmental performance and solving many of the environmental problems that appear within the designing stage.

### 1.2 Research Aims

The study aims to analyse and measure the connection between mathematics and environmental architecture performance by:

- Illustrating mathematical branches and identifying their application in architecture.
- Clarifying environmental performance concept and evaluation criteria.
- Prove that mathematics has a vital role in promoting the building's environmental performance.

### 1.3 Research Methodology

For attaining the research aims, the methodology was divided into three parts; ***First,*** clarify mathematics different fields and their application in architecture, along with identifying environmental performance concepts and its evaluation criteria. ***Second***, analyse several architectural projects that effectively optimize their environmental performance by using mathematics to investigate to what extent the mathematical fields could promote the environmental performance. ***Third,*** apply one of the mathematical optimization techniques on a virtual prototype to environmentally optimize its efficiency and simulate its environmental performance to validate the mathematics capability in improving environmental performance.

## 2. Mathematics in Architecture.

### 2.1 Historical Review

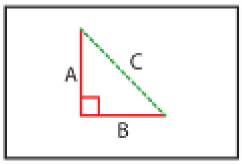
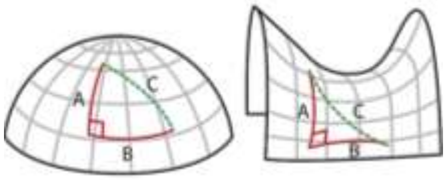
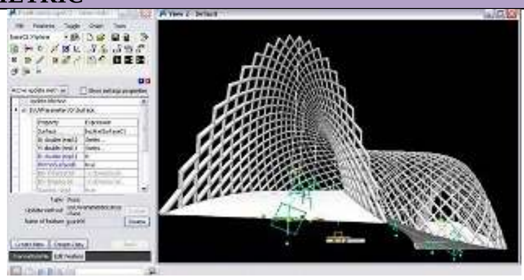
Mathematics has always existed in creating art and architecture throughout history. By starting with geometry, it's roots could be seen in Egyptian, Babylonian, and Hindu old civilizations [5]. As a result, geometry's types could represent entire periods in architecture as; Euclidean geometry from Antiquity to the Romanesque period, Perspective and Projective geometry from the Gothic and Renaissance period to the Neo-classical eras, and non-Euclidean geometries for modern architecture [6].

Whereas, in the digital era with the integration between mathematics and computer techniques, new mathematical tools have renovated with more architectural benefits, rather than just being for modelling and presenting, it had a main role in form-finding, shaping, generating of forms, as in parametric and algorithmic generation, and topological formations. Subsequently, it opened new horizons for mathematical and geometrical contribution in architecture, [2,7].

### 2.2 Mathematical Branches

There were several mathematical branches that helped in forming architecture. The paper will discuss four of the commonly used branches, which are; *Geometry*, *Parametric*, *Algorithm*, and *Topology*. Table 1 shows the differences between previous branches and their application in architecture.

**Table 1 Types of Mathematics in Architecture**

<table>
<tr><th colspan="4">MATHEMATICS TYPES</th></tr>
<tr><th colspan="4">GEOMETRY</th></tr>
<tr><td colspan="4">Geometry is the fundamental science of forms. It studies their order, proportions, angles and transformations [8].</td></tr>
<tr><td rowspan="4">Geometrical Branches</td><td>Euclidean Geometry</td><td>- It was discovered by Euclid in 300 B.C [9].<br>- It deals with points, lines, planes, and other geometric figures in the planar case, Fig 1 and based on Euclid basic five Postulates [9].</td><td><br><br>Figure 1 The planar case concept of Euclidean geometry 10]</td></tr>
<tr><td>Non-Euclidean Geometry</td><td>- Hyperbolic (developed by Nikolai Lobachevsky in 1829 and Janos Bolyai in 1832). While elliptical (developed by Bernhard Riemann in 1867) [11].<br>- Hyperbolic deals with surfaces with negative curvature, elliptical with positive curvature ones [9], Fig 2.</td><td><br><br>Figure 2 The elliptical and hyperbolic concept of non-Euclidean geometry [10]</td></tr>
<tr><td>Discrete Differential Geometry (DDG)</td><td>- It began in late of 20th century by Robert Sauer and Walter Wunderlich, then was continuously developed by scientist [12].<br>- It discrete freeform structures while preserving its global geometric and structural properties, to facilitate designing and fabrication [13], Fig 3.</td><td><br>Figure 3 The concept of DDG [13]</td></tr>
<tr><td>Fractal Geometry</td><td>- It was developed in 1975, by Benoit Mandelbrot.<br>- It describes irregular structures, that formed from self-similar elements, as in Peano Curve and Sierpinski Triangle [14], Fig 4.</td><td><br>Figure 4 Sierpinski Triangle [15]</td></tr>
<tr><th colspan="4">ALGORITHM</th></tr>
<tr><td colspan="3">- The word Algorithm is based on a concept attributed to Al-Khwarizmi in the 8th century [16].<br>- It is a finite set of rules used to achieve a definite objective in a fixed number of steps. By taking one or a set of values as input, performs a sequence of computational steps to generate one or a set of values as output [17], Fig 5.</td><td><br>Figure 5 Algorithm Script in Grasshopper [Author]</td></tr>
<tr><th colspan="4">PARAMETRIC</th></tr>
<tr><td colspan="3">- The term 'Parametric' originates from mathematics “Parametric equation” which refers to using certain variables or parameters that can be edited to modify the output of an equation [1].<br>- Parametric design process comprises of variable attributes “parameters” and fixed attributes “constraints” [18]. Varying the attributes generate multiple design alternatives each called “instance [19], Fig 6.</td><td><br>Figure 6 3D Parametric instance in Generative Components [20]</td></tr>
</table>

| TOPOLOGY | |
|---|---|
| - The first appearance of topological approach in architecture was in Greg Lynn essay "Folding in Architecture" in 1993, inspired by Gilles Deleuze's philosophical theory "fold" [21].<br>- It is the transformations of one object to another, without cutting or splicing, while maintaining its topological identity [9], Fig 7. | <br>**Figure 7 The Pretzel Transformation [22]** |

## 3. Environmental Performance.

Environmental performance is one of the main categories in architecture performance, as being performative is usually related to sustainability and environmental behaviour of buildings [23].

### 3.1 Environmental Performance Evaluation Criteria

Environmental performance criteria could be classified in to two main sections; *functionality and efficiency*, each section has sub-categories [24]. Table 2 illustrates each of them in detail.

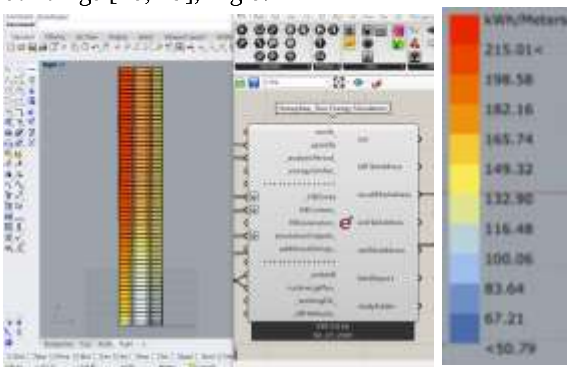
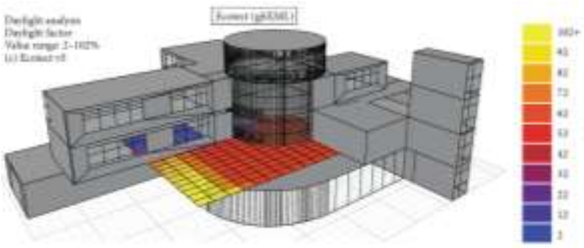
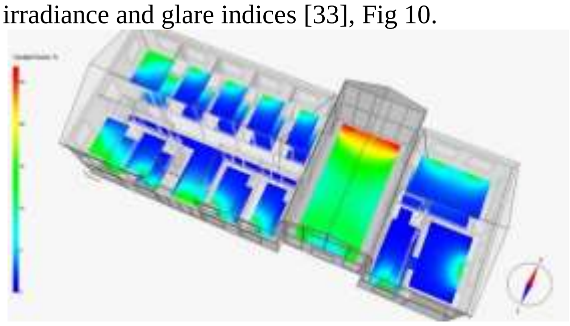
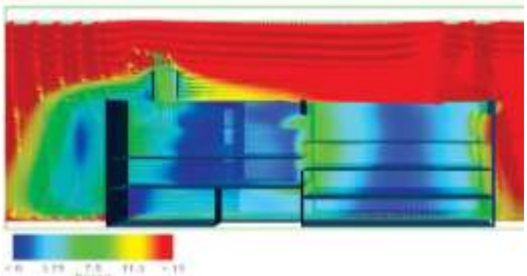
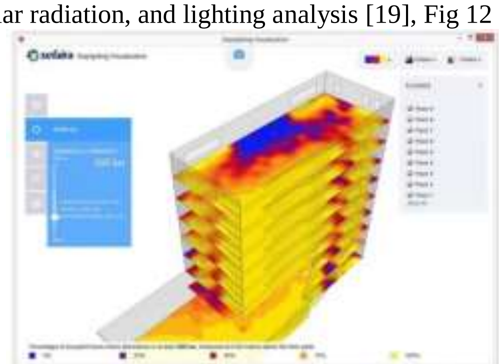
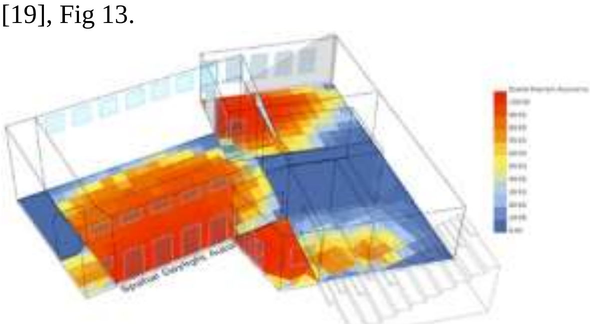
**Table 2 Environmental Performance Evaluation Criteria** [24, 25, 26, 27, Author]

| | | | |
|---|---|---|---|
| **Environmental Performance Evaluation Criteria** | **Functionality** | **Adequate Space** | help in providing suitable space for the required functions |
| | | **Adjacencies of Functionality Related Areas** | indicates the direct connection between relative functional areas |
| | | **Amount and Distribution of Square Footage** | means the providing of suitable area corresponding to user's amount and distribution in the space |
| | **Efficiency** | **Energy Efficiency** | seeks to reduce energy consumption, carbon emissions, and passive heat gain. Also, utilize and produce renewable energy, provide energy monitoring, by using energy efficient equipment, to optimize energy performance. |
| | | **Material Efficiency** | aims to use regionally, renewable, lightweight, prefabricated, and recycled materials, to manage construction and demolition waste, design for durability, flexibility, and reduce building life-cycle impact. |
| | | **Thermal Efficiency** | intends to promote user's comfort, productivity, and well-being by providing a thermally comfortable environment. |
| | | **Air Efficiency** | search for minimizing air pollution, by minimizing indoor air pollution, provide natural ventilation, use low-emitting materials, and promote air quality within design, specification, installation, and occupancy processes. |
| | | **Light Efficiency** | seeks to promote visual performance, productivity, and comfort by enhancing natural daylighting, reducing artificial lighting and respond to occupant demands early at the design stage. |
| | | **Acoustics Efficiency** | promotes occupants' productivity, and communications through effective acoustic performance that met the standards and testing requirements, by using of effective sound insulation and noise control strategies, reduce sound transmission and reverberation times. |

### 3.2 Environmental Performance Computational Simulations

Recently, building performance simulations (BPS) became an integral part in designing and evaluating building's environmental performance. So, they enabled designers to simulate and quantify the impacts of various design strategies that affect building environmental performance early in the design process. By analysing different environmental criteria as; energy consumption, daylighting, natural ventilation, or solar radiation [19]. Table 3 shows a few of the commonly used environmental simulation Software:

**Table 3 Examples of Environmental Analysis Software and Tools**

| EnergyPlus | Autodesk Ecotect |
|---|---|
| EnergyPlus is a simulation engine, that reads input and writes output to text files based on a user's description of a building from. it is used to analyse energy consumption (thermal, heating, cooling, ventilation, and lighting loads), and water use in buildings [28, 29], Fig 8.<br><br>**Figure 8 EnergyPlus cooling energy simulation in Rhino [Author]** | Ecotect is an environmental analysis tool that is embedded to Autodesk CAD, and compatible with Autodesk REVIT. Ecotect analyses different building performance, as solar heat gain, shading, lighting and daylighting studies, energy consumption, and natural airflow for ventilation [30, 31], Fig 9.<br><br>**Figure 9 Ecotect Daylight analysis [32]** |
| **Radiance** | **Computational Fluid Dynamics (CFD)** |
| Radiance is an analysis tool that uses input files to specify the geometry as materials, luminaires, time, date and sky conditions, to analyse daylighting, illumination, visual quality, spectral radiance, irradiance and glare indices [33], Fig 10.<br><br>**Figure 10 Radiance Daylight Simulation Results [34]** | CFD is a tool that uses algorithms and numerical methods to solve and analyses mass, momentum, thermal energy, natural ventilation design, building material emissions, complex flows of fire and smoke, and predictions of noise [35], Fig 11.<br><br>**Figure 11 CFD Simulation and Analysis Model [35]** |
| **Insight 360 and Sefaira** | **Honeybee and Ladybug** |
| Insight 360 and Sefaira are plugins for Revit. Sefaira is used in performing energy and daylight analysis. While, Insight 360 is used in performing energy, solar radiation, and lighting analysis [19], Fig 12<br><br>**Figure 12 Sefaira Daylight Analysis [36]** | They are both open-source plugins for Grasshopper and Rhino. Honeybee can perform energy, daylight, natural ventilation, and heat transfer analysis. While, Ladybug performs sun path, shadow range, solar radiation, outdoor comfort, and wind roses analysis [19], Fig 13.<br><br>**Figure 13 Honeybee Daylight Analysis [37]** |

## 4. Mathematics, Architecture and Environmental Performance.

Mathematics and architecture have always been related; the both roots are embedded in geometry. Architecture has been concerned with the creation of space; mathematics with its description and definition. In architecture, mathematical rules and techniques were used to offer solutions to the problems of defining and building free-form surfaces [9].

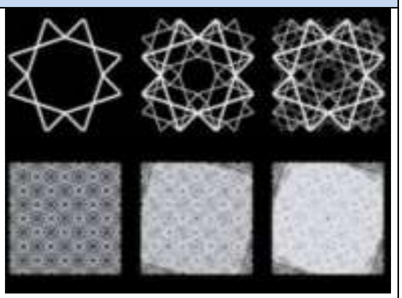
In the last two and half decades, Digital computation has given architects new creative opportunities to creatively design and optimize geometrical space, that was opened up by mathematics. In general, mathematical computation techniques are used in a verity of ways to solve spaces problems or optimize their performance, as structural optimization as resolving the panelization of surface, or environmental as resolve the space's acoustic efficiency [9].

The following part of the research will discuss the relationship between each branch of mathematics and architecture in detail and their role in promoting building environmental performance.

### 4.1. Geometry

Geometry has an important role in developing architectural forms and spaces, as it studies the environmental, aesthetical and structural characteristics of the geometrical figures and shapes. In addition to, their proportions, angles, relations, differences, and transformations. Accordingly, geometry can facilitate selecting the shapes in initial design phase and indicate which have the higher performance, and help in developing these shapes to further promote building environmental performance, as creating suitable spaces for the required functions, optimizing material distribution, and reduce energy consumption, and material usage [1, 14]. That can appear clearly in the following two examples, Table 4.

**Table 4 Geometry Application in Promoting Environmental Performance**

| **The Shell House (Japan, 2008)** | | |
|---|---|---|
| *The Shell House*, Fig 14, used elliptical geometry to integrates the project with its surrounding environment. The house was formed from two elliptical intertwined tubes, centralized around a large fir tree, creating an organic form that integrate with its surrounding nature [14]. |  **Figure 14 The Shell House [39]** | |
| **Louvre Abu Dhabi (UAE, 2017)** | | |
| *Louvre Abu Dhabi*, Fig 15, used fractal geometry in generating (Islamic star patterns), with different order and scale and utilize it to provide shade, optimize thermal efficiency, minimize solar gain, and energy consumption [9, 38], Fig 16. |  **Figure 15 Louvre Abu Dhabi [40]** |  **Figure 16 The Louvre's fractal star patterns [41]** |

In architecture, Geometry importance locates in its ability in developing geometrical forms and patterns by utilizing the form's characteristics, properties, proportions, order, scale and transformation to promote building's environmental performance efficiency.

### 4.2. Algorithm

Algorithmic design or Generative design, is a programming-based design approach [42], allows designers to script the design process in a set of instructions, as environmental or functional requirements, then generate, evaluate and filter the outputs to chooses the optimal solution from a set of generated alternatives [9, 11]. Algorithmic Generative design concept can be clarified in two of its techniques; L-System and Genetic Algorithms (GA).

L-system, is an algorithmic generative technique that produce fractal's structure describes the growth process and structure characteristics of plants and natural forms [43, 44].

Genetic Algorithm is an evolutionary algorithm resembles the biological evolution processes, in which the best characteristics are continuously selected and transferred to the next generations, so at the end of the process, the last generated forms will have the best characteristics [46].

The two techniques concepts can be cleared in the following two case studies, L-system in The Tote Restaurant and Genetic Algorithm in Alibaba's Shanghai offices, Table 5.

**Table 5 Algorithm Application in Promoting Environmental Performance**

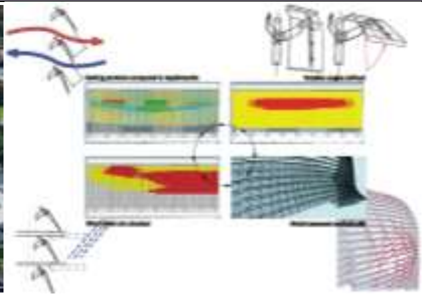
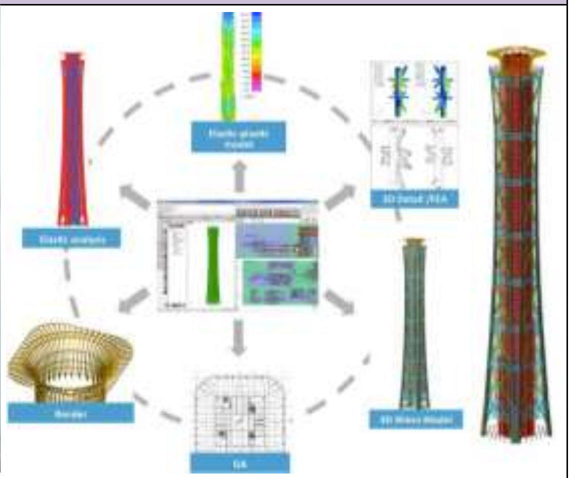
| **The Tote Restaurant (India, 2009)** | | |
|---|---|---|
| *The Tote Restaurant*, Fig 17. The designers used L-system technique to design steel intersecting columns resembles the branching trees around the building, Fig 18, as a preservation of Mumbai historic buildings and the green areas surrounding the building and to integrate the structure with its surrounding environment [14]. |  **Figure 17 The Tote Restaurant [45]** |  **Figure 18 The branching column [45]** |
| **Alibaba's Shanghai offices (Foster + Partners, 1st prize of Alibaba's offices competition 2020)** | | |
| Alibaba's Shanghai offices, Fig 19. GA was used to design the optimal office formation. Several requirements were coded as; area requirements, maximize outside views, visual connectivity, user comfort, protection from winds and summer sun, natural lighting and low construction wastage. The coded requirements were used to generate different alternatives, analyzed and filter them till reach the optimal solution [47], Fig 20. |  **Figure 19 Alibaba's Shanghai offices [47]** |  **Figure 20 Alibaba's Shanghai offices interior space for visual connectivity [47]** |

In architecture, Algorithm capability locates in coding building's requirements whether environmental or structural requirements, then generate and evaluate, and filter multiple design solution in continuous cycle till reach the most performative solution.

### 4.3. Parametric

Parametric is a computational generative and analytical method [17], that use variable and fixed parameters to generate multiple design alternatives each by changing these parameters [18, 19]. Parametric design can model and generate complex architectural forms with few parameters, as environmental or functional requirements [48]. In addition, Parametric design has the ability to integrate with *Building Performance Simulations (BPS)* and *Evolutionary Optimization Algorithms* to increase its capacity to orient the design process to the optimal performative solution [19, 49]. Parametric capabilities can be seen in the following examples, Table 6

**Table 6 Parametric Application in Promoting Environmental Performance**

| **Aviva Stadium (Ireland, 2010)** | | |
|---|---|---|
| Aviva Stadium, Fig 21, used a single parametric model to design the overall geometrical form. Parametric design had analyzed and tested the position, rotation values and opening angle of the façade panels to provide natural air ventilation, remove hot air behind the panels, and minimizing the effect of windblown rain, and optimize building environmental performance [17, 50], Fig 22. |  **Figure 21 Aviva Stadium [51]** |  **Figure 22 Aviva Stadium [50]** |
| **CITIC Tower (China, 2018)** | | |
| CITIC Tower, Fig 23, used parametric framework to reach the optimal balance between design formation, functional efficiency and structural performance. Several design alternatives were generated, in the initial design stage, with different waist heights, planar sizes, and functional zonings. The alternatives were analysed and compared continuously till achieve optimal aesthetical and structural performance. Parametric design helped in optimized floor usage efficiency by providing more office spaces at the higher levels, comfortable working space, easy access; and minimize walking distance, Fig 24. [52, 53, 54, 55]. |  **Figure 23 CITIC Tower [55]** |  **Figure 24 the total parametric framework of Analysis and Design Engine [55]** |

In architecture, *Parametric design* is a powerful tool that has the ability to model generate and simulate complex architectural forms based on the project's requirements as environmental criteria, early in the design process, to promote its building's efficiency.

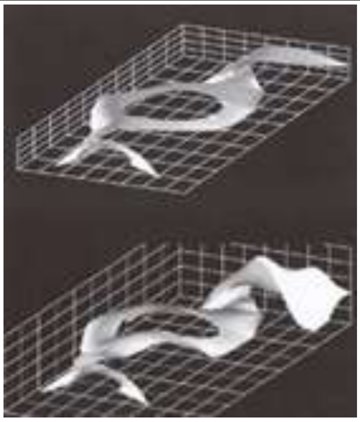
In addition, Parametric has the capability to integrate with genetic optimization algorithms that analyse the different generated alternative in continuous cycle till select the optimal alternative.

Further, it has ability to integrate with BPS that connect the conceptional model with external simulation software whether structural or environmental analysis software to simulate building's performance values under structural load or external environment to promote building performance efficiency within design stage.

## 4.4. Topology

Topology Optimization (TO) is an algorithmic-based approach that mimic the rules of biological growth in nature, to promote architecture performance. It utilizes optimization algorithms to find the optimal topological configurations that have the optimal structural efficiency, optimal material density and distribution, and least material usage and wasted energy, within a given design requirements [56]. Topology Optimization generates strong organic architectural forms with optimal structural response to applied loads, in a similar way as forms in nature [57]. Its techniques can be clarified in the following examples, Table 7.

**Table 7 Topology Application in Promoting Environmental Performance**

<table>
<tr><td colspan="3">Grin Grin Park (Japan, 2005)</td></tr>
<tr><td>Grin Grin Park, Fig 25. The Park greenhouses have complex freeform wavy shape, merges the greenhouses with their surrounding landscape. Topology Optimization technique was used from the initial form finding process to optimize the park's geometrical formation, achieve optimal structure efficiency, and enclose maximum volume with minimum material [9, 57], Fig 26.</td><td><br>Figure 25 Grin Grin Park [9]</td><td><br>Figure 26 The TO Form Evolution of Grin Grin Park [9]</td></tr>
<tr><td colspan="3">Qatar National Convention Centre (Qatar, 2011)</td></tr>
<tr><td>Qatar National Convention Centre, Fig 27. The Centre entrance design has a dominant intertwined trees-like structure support the building's exterior canopy. The intertwined trees are symbol for Sidra tree in Qatari culture. The intertwined trees-like structure was shaped using EESO [9]. EESO is a topology optimization method that uses stress contour lines and surfaces in repeated shape analysis process to generate optimal structural shape that uses the least possible material [58], Fig 28.</td><td><br>Figure 27 Qatar National Convention Centre [58, 9]</td><td><br>Figure 28 The EESO Form Evolution [58, 9]</td></tr>
</table>

In architecture, Topology optimization is mainly use in shaping architectural forms to have the ideal topological configurations, the optimal structural efficiency, optimal material density and distribution, and least material usage and wasted energy. Thus, environmentally it mainly helps in decreasing material usage and wasted energy in construction process.

## 5. Applied Study

The applied study aims to clarify the effect of using mathematics early in the design process in promoting building's environmental performance.to

### 5.1 The Methodology.

Therefore, to accomplish the applied study goal, the study was parted into two processes;

- ***First, Generation process:*** an algorithmic generative function was used on a basic geometrical formation (a vertical square tower) to indicate its optimal deformation that minimizes the solar heat gain on its envelope to the lowest value. Thus, heat gain was chosen as it directly influences many environmental performances criteria as energy, thermal, and air efficiency.
- ***Second, Evaluation process*****:** an energy, thermal, and air temperature simulation analysis were performed on both the basic and optimized formation to compare and verify the efficiency of the optimized solution.

The two processes were performed by using ***Rhino Software*** and ***Grasshopper***, ***Ladybug***, and ***Honeybee plugins***.

### 5.2 The Processes.

- ***First, Generation Process:***

The study started by a basic square geometrical form, as an office tower with 52 floors locates in Cairo- Egypt. So, to generate the optimized form number of steps were performed, as follows:

- *The basic geometry design;* originally the square tower was oriented by consider two sides faced north for promoting light and air efficiency. Then, the area of the lowest 10 floors were maximized to increase the tower's stability, Fig 29.
- *The basic form solar radiation analysis*; later, an annual solar radiation analysis was performed on the tower to indicate its solar heat gain coefficient, Fig 30.

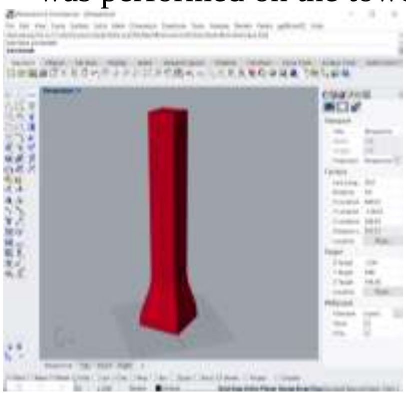


**Figure 29 The basic geometry [Author]**

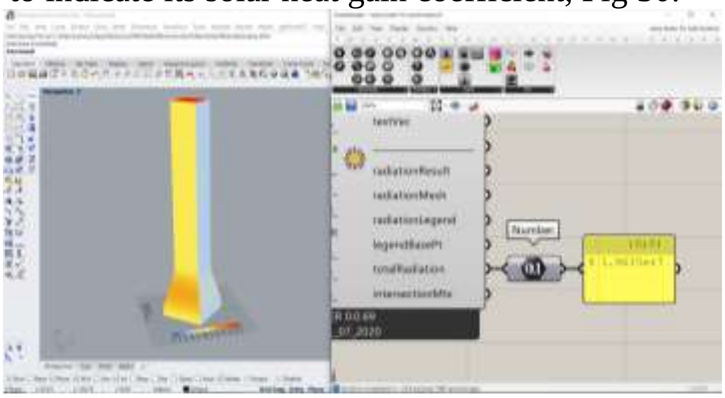
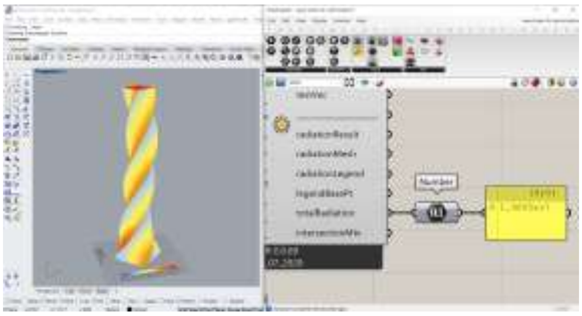
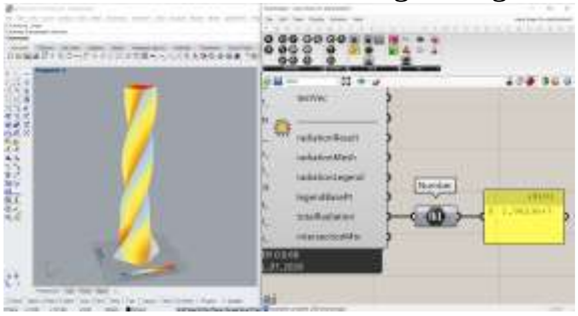


**Figure 30 The solar heat gain value of the basic geometry: 1.9615 kWh [Author]**

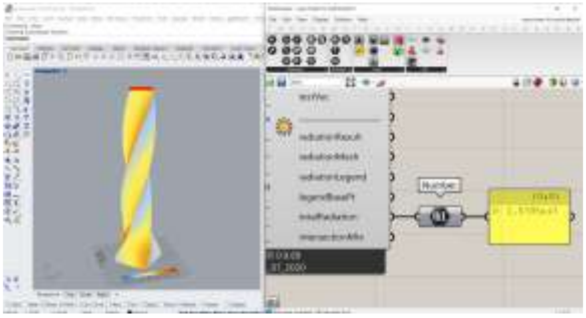
- *Alternatives generation;* next, an algorithmic generative component (Galapagos component) was used to generate different design alternative for the basic geometry with different rotation according to the lowest solar radiation value. It has generated 2601 alternative, evaluated, and filtered them to find the optimal solution.
- *The optimal alternative; finally,* Galapagos component showed that the optimal solution has 1.9447 kWh solar radiation value and the lowest solar heat gain, Fig 31.



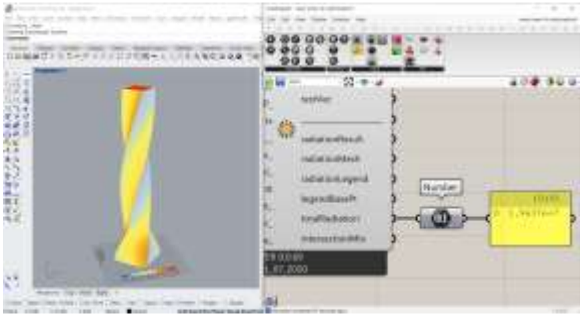
a)*The highest solar radiation value 1.9895 kWh*



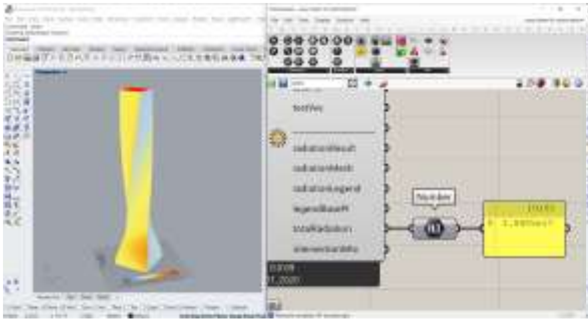
b)*Design alternative with solar radiation value 1.9823 kWh*



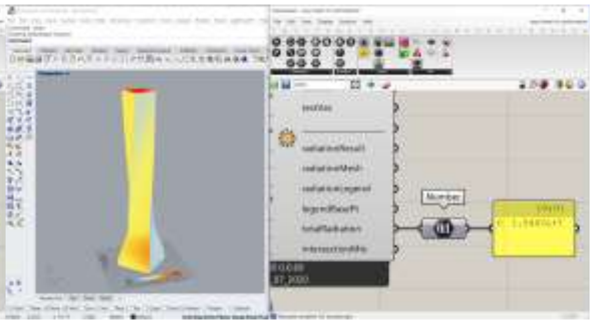
c)*Design alternative with solar radiation value 1.9705 kWh*



d)*Design alternative with solar radiation value 1.9637 kWh*



e)*Design alternative with solar radiation value 1.9485 kWh*



f)*The optimal solution with the lowest solar radiation value 1.9447 kWh*

**Figure 31 The solar radiation value of the different generated alternatives – organized from the highest to the lowest value [Author]**

- ***Second, Evaluation process***

Consequently, before comparing the environmental performance of the basic and selected geometries, they were first adjusted to confirm the result accuracy. The both geometries were divided into 52 floors with central core, then cladded with glass panels where the glazing ratio is 0.75% in the overall geometry, Fig 32, 33.

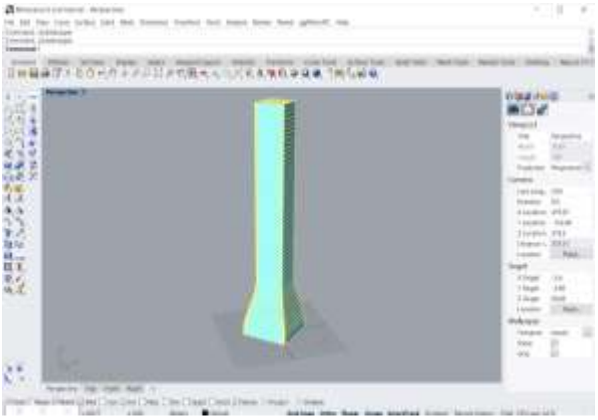


**Figure 32 The main geometry [Author]**

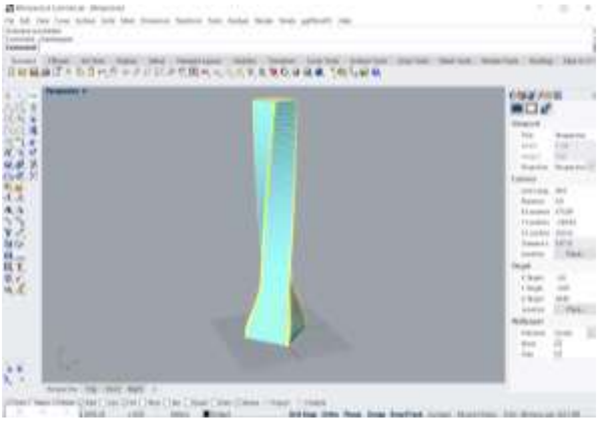
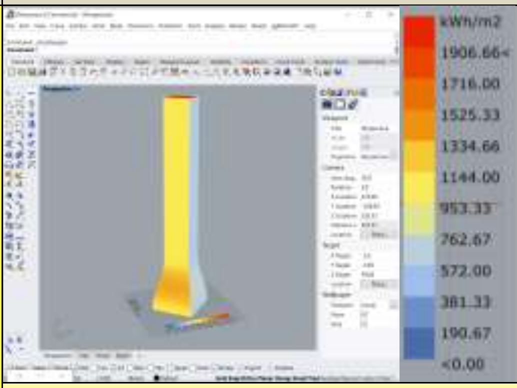
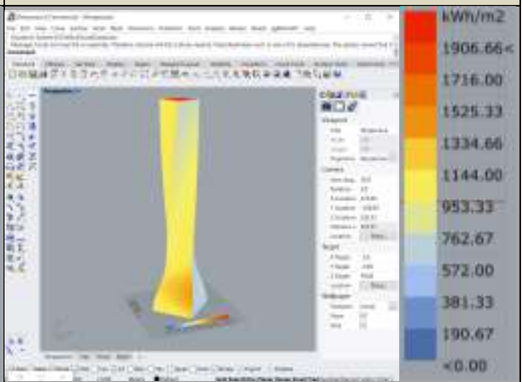
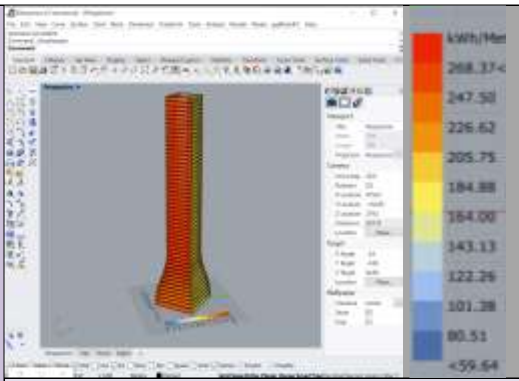
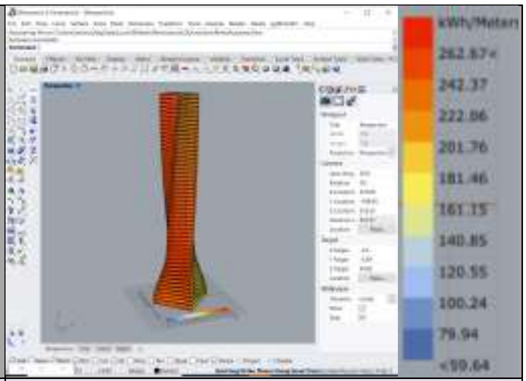
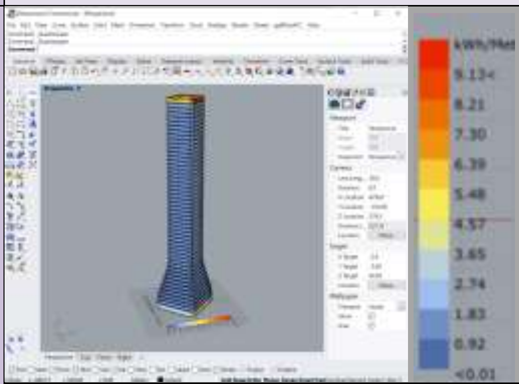
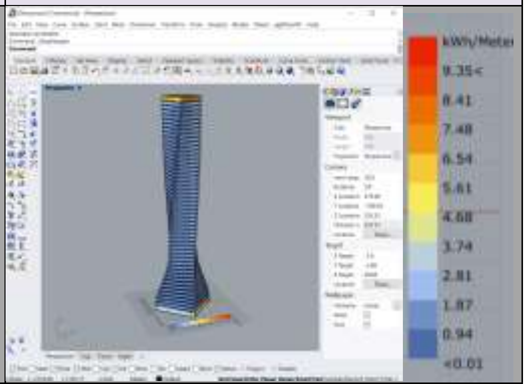
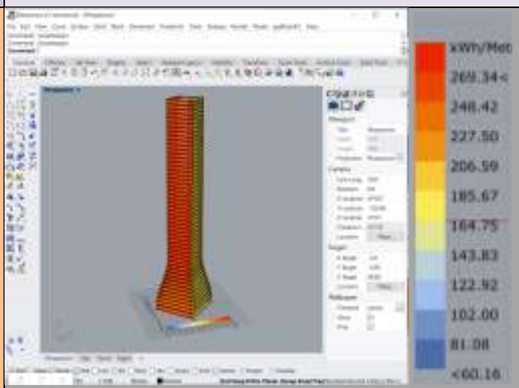
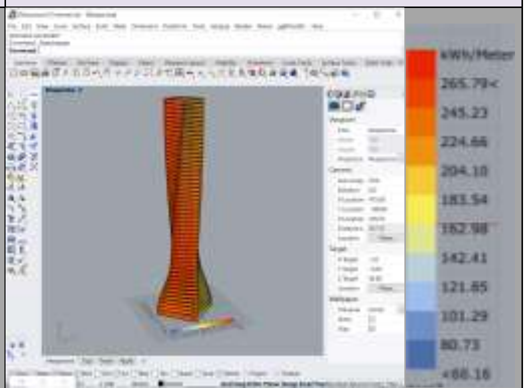


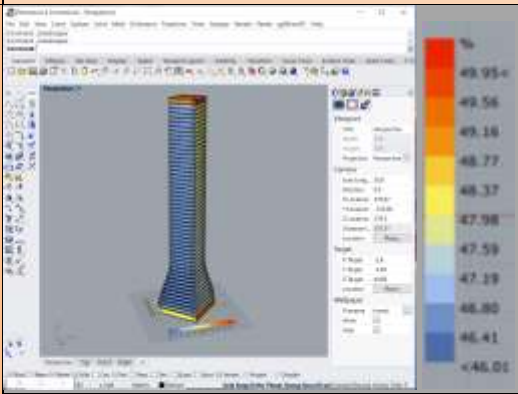
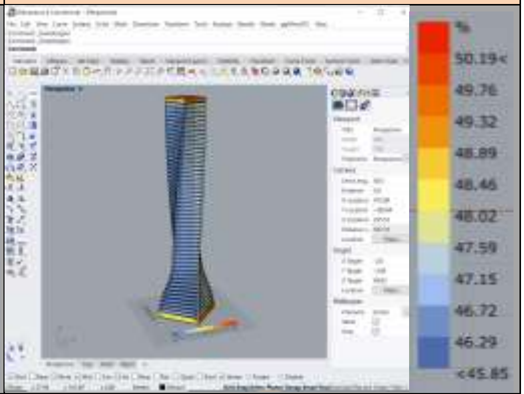
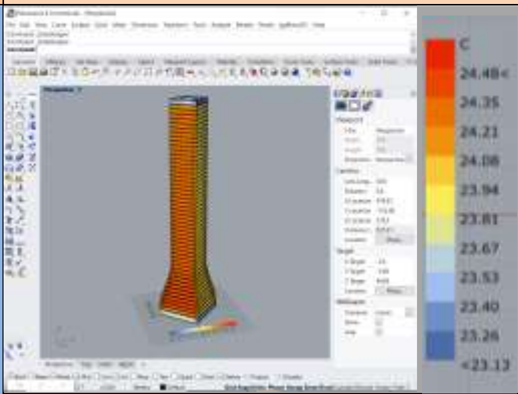
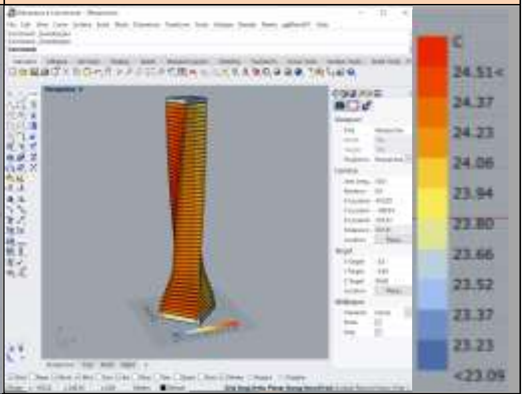
**Figure 33 The optimized geometry [Author]**

Thus, Table 4 shows a comparison between the environmental performance of the main and optimized geometry. The comparison started with comparing the two geometries' *Solar Heat Gain*, Fig 34, 35. Then, their *Energy, Thermal, and Air Temperature* simulations results to compare their performance, Fig 36 to Fig 45.

**Table 4 Comparison Between the Environmental Performance of Main and Optimized Geometry [Author]**

| Environmental Performance | The Main Geometry | The Optimized Geometry |
|---|---|---|
| **Average Annual Solar Heat gain (kWh/m2)** |   **Figure 34 The Main Geometry Average Annual Solar Heat gain** |   **Figure 35 The Optimized Geometry Average Annual Solar Heat gain** |
| **North-east facade** | 572 kWh/m$^2$ | 381.3 kWh/m$^2$ |
| **North-west facade** | 572 kWh/m$^2$ | From 953.3 kWh/m$^2$ in upper floors to 572 kWh/m$^2$ in lower floors |
| **South-east facade** | From 953.3 kWh/m$^2$ in upper floors to 1334.6 kWh/m$^2$ in lower floors | From 572 kWh/m$^2$ in upper floors to 1334.6 kWh/m$^2$ in lower floors |
| **South-west facade** | From 953.3 kWh/m$^2$ in upper floors to 1334.6 kWh/m$^2$ in lower floors | From 953.3 kWh/m$^2$ in upper floors to 1525.3 kWh/m$^2$ in lower floors |

| | | | |
|---|---|---|---|
| **Energy Efficiency** | **Average Annual Cooling Energy Load Per Floor (kWh/m2)** | <br>**Figure 36 The Main Geometry Average Annual Cooling Energy Load Per Floor** | <br>**Figure 37 The Optimized Geometry Average Annual Cooling Energy Load Per Floor** |
| | **North-east Zone** | 164 kWh/m$^2$ | 161.1 kWh/m$^2$ |
| | **North-west Zone** | 164 kWh/m$^2$ | From 262.6 kWh/m$^2$ in upper floors to 161.1 kWh/m$^2$ in lower floors |
| | **South-east Zone** | 226.6 kWh/m$^2$ | From 181.4 kWh/m$^2$ in upper floors to 222 kWh/m$^2$ in lower floors |
| | **South-west Zone** | 247.5 kWh/m$^2$ | From 222 kWh/m$^2$ in upper floors to 242.3 kWh/m$^2$ in lower floors |
| | **Average Annual Heating Energy Load Per Floor (kWh/m2)** | <br>**Figure 38 The Main Geometry Average Annual Heating Energy Load Per Floor** | <br>**Figure 39 The Optimized Geometry Average Annual Heating Energy Load Per Floor** |
| | **North-east Zone** | From 2.7 kWh/m$^2$ in upper to 1.8 kWh/m$^2$ in lower floors | From 2.8 kWh/m$^2$ in upper to 1.8 kWh/m$^2$ in lower floors |
| | **North-west Zone** | From 2.7 kWh/m$^2$ in upper to 1.8 kWh/m$^2$ in lower floors | 1.8 kWh/m$^2$ |
| | **South-east Zone** | From 1.8 kWh/m$^2$ in upper to 0.9 kWh/m$^2$ in lower floors | From1.8 kWh/m$^2$ in upper to 0.9 kWh/m$^2$ in lower floors |
| | **South-west Zone** | From 1.8 kWh/m$^2$ in upper to 0.9 kWh/m$^2$ in lower floors | From 1.8 kWh/m$^2$ in upper to 0.9 kWh/m$^2$ in lower floors |
| **Thermal Efficiency** | **Average Annual Thermal Load Per Floor (kWh/m2)** | <br>**Figure 40 The Main Geometry Average Annual Thermal Load Per Floor** | <br>**Figure 41 The Optimized Geometry Average Annual Thermal Load Per Floor** |

| | | | |
|---|---|---|---|
| | **North-east Zone** | 164.7kWh/m$^2$ | 162.9 kWh/m$^2$ |
| | **North-west Zone** | 164.7kWh/m$^2$ | From 265.7 kWh/m$^2$ in upper floors to 162.9 kWh/m$^2$ in lower floors |
| | **South-east Zone** | 227.5 kWh/m$^2$ | From 183.5 kWh/m$^2$ in upper floors to 224.6 kWh/m$^2$ in lower floors |
| | **South-west Zone** | 248.4 kWh/m$^2$ | 245.2 kWh/m$^2$ |
| | **Average Annual Relative Humidity (%)** |  **Figure 42 The Main Geometry Average Annual Relative Humidity** |  **Figure 43 The Optimized Geometry Average Annual Relative Humidity** |
| | **North-east Zone** | From 47.9% in upper floors to 47.1% in lower floors | From 48% in upper floors to 47.5% in lower floors |
| | **North-west Zone** | From 47.9% in upper floors to 47.1% in lower floors | From 47.1% in upper floors to 47.5% in lower floors |
| | **South-east Zone** | From 47.1% in upper floors to 46.4% in lower floors | From 47.5% in upper floors to 46.2% in lower floors |
| | **South-west Zone** | From 47.1% in upper floors to 46.4% in lower floors | From 46.7% in upper floors to 46.2% in lower floors |
| **Average Annual Air Temperature (C°)** | |  **Figure 44 The Main Geometry Average Annual Air Temperature** |  **Figure 45 The Optimized Geometry Average Annual Air Temperature** |
| **North-east Zone** | | 23.8 C° | From 23.6 C° in upper floors to 23.8 C° in lower floors |
| **North-west Zone** | | From 23.8 C° in upper floors to 23.9 C° in lower floors | From 23.8 C° in upper floors to 23.9 C° in lower floors |
| **South-east Zone** | | From 24 C° in upper floors to 24.3 C° in lower floors | From 23.8 C° in upper floors to 24.2 C° in lower floors |
| **South-west Zone** | | From 24 C° in upper floors to 24.5 C° in lower floors | From 24 C° in upper floors to 24.3 C° in lower floors |

### 5.3 Results and Discussion

The applied study has used parametric design and genetic algorithm as tools to promote building performance. they were used to generate different design alternatives, analysed and filter them, then select the optimal solution with the highest environmental performance, then both the basic and the optimized model were environmentally analysed to verify the optimized solution efficiency, and as a result, by comparing the result of the two models' *Energy, Thermal, and Air Temperature* simulations, the optimized geometry has shown higher environmental performance efficiency, as:

- *The average annual solar heat gain* was decreased on the north-east facade by 34%, while on the south-east façade's upper floors by 40%.
- The *average annual cooling energy load per floor* was decreased in the south-east zones of the upper floors by 20%, and the south-west zones by 10 %.
- The *average annual heating energy load per floor* was decreased in the north-west zones of the upper floors by 34%.
- The *average annual Thermal load per floor* was decreased in the south-east zones of the upper floors by 19%.
- *The Average Annual Air Temperature* optimization ratio was low compared to other performances as the tower was modelled to have HVAC system. So, despite that the air temperature ratio is almost the same, the required cooling and heating energy load per floor was decreased by high percentage.

## 6. Conclusion and Recommendation

Accordingly, from the theoretical, analytical, and applied study of utilizing mathematics in promoting environmental performance efficiency, some points were concluded followed by some recommendations.

### 6.1 Conclusion

The present study determined the effect of utilizing mathematics fields as a design tool to promote environmental Performance. Results have shown that;

1. The four branches (*Geometry, Parametric, Algorithmic and Topology*) are used effectively for promoting environmental performance in contemporary architecture.
2. Both functionality and efficiency factors could be attained by using different mathematical fields.
3. *Geometry* is mainly used in developing architectural forms by utilizing the properties, proportions, order, scale and transformation of geometrical shape to promote building environmental performance.
4. *Generative Algorithmic Design* allows designers to code any environmental requirements within the design process, then generate multiple solution, evaluate and filter them many times till reach optimal performative solution.
5. *Parametric Design* has great potential in generate different geometrical alternatives early in the design process, based on the project's environmental criteria.

6. *Parametric Design and Genetic Algorithmic Design* can be integrated to orient the design process to the optimal performative solution early in form-finding process.
7. *Parametric Design* can integrate with building performance simulations (BPS) to measure the efficiency of the multiple generated solution to facilitate promoting project's environmental efficiency.
8. *Topology Optimization techniques* are used to find building's optimal topological configurations that have the optimal structural efficiency, and use the least material, within a given design requirements.
9. One or more mathematical branch could be merged and used in same project design process to upgrade the environmental performance efficiency.
10. Mathematics helped in developing creative environmental solutions, and promoting building's efficiency.

## 6.2 Recommendations and Future Work

1. Further studies in this field to explore mathematical branches role in promoting other architectural performance categorization as structural and aesthetical performance.
2. Studying each mathematical branch techniques and methods in promoting building's architectural performance.
3. Architecture Students must be supplied, and learn how to use advanced architecture software that based on mathematics scripting which had a great role increasing their creativity, and abilities, and skills.

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