Using An accelerated Technique of The Laplace-Adomian Decomposition Method in Solving A class of Non-linear Integro-differential Equations

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Abstract

In this work we use an efficient technique based on Adomian Decomposition Method (ADM) and Laplace Transform to solve non-linear integro-differential equations. This method effectively handles non-linear integro differential equations of the first and the second kind. Finally, some examples will be discussed to support the proposed method.

Keywords: Laplace Adomian Decomposition Method, Accelerated Adomian polynomials, Traditional Adomian Polynomials, integro-differential equations.

1 Introduction

It is well known that linear and non-linear Volterra integral equations arise in many scientific fields such as the population dynamics, spread of epidemics, and semi-conductor devices. Volterra started working on integral equations in 1884, but his serious study began in 1896. Hence, there are numerous researchers that interested in studying this type of equations. A comparison was made between Adomian decomposition and tau methods in [1] for finding the solution of Volterra integro-differential equations. In [2] the author used a combined form of the Laplace transform method with the Adomian decomposition method to get the analytic solution of the non-linear Volterra integro–differential equations of first and second kind. Rashed [3] studied integro–differential equations depending on Lagrange interpolation. The author [4] in studied by using Wavelet–Galerkin method for solving integro–differential equations. Wazwaz [5] used a variety of powerful

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methods in solving a non-linear integral and integro- differential equations and for making a comparative study.

Laplace transform technique in combination with Adomian decomposition method is presented and modified, which was first studied by Khuri in [6] to solve non-linear differential equations. In [7], the authors investigated the method for solving coupled non-linear partial differential equations. Elgasery [8] applied the Laplace decomposition method for the solution of Falkner–Skan equation which describes two dimension incompressible laminar boundary layer equations. Laplace decomposition method was employed to logistic differential equations to find the numerical solutions in [9]. Chanquing and Jianhua studied the Adomian decomposition method to solve the non-linear fractional differential equations in [10].

In [11], the technique was applied on delay differential equations. In [12], the author used a modified Laplace Adomian decomposition method (LADM) to solve the integro-differential equations. To overcome the non-linearity term used the traditional Adomian polynomials. Magdy and Mohamed [13] practiced Laplace decomposition method and Pade approximation to get the numerical solution of non-linear system of partial differential equations. Further, a modified Laplace decomposition method was adopted for Lane-Emden type differential equations in [14]. The author [15] used a combined form of the modified Laplace Adomian decomposition method (LADM) to get the analytic solution of the non-linear Volterra-Fredholm integro differential equations of the first and second kind.

In this paper, we present Laplace Adomian Decomposition Method for solving the non-linear integro-differential equations in the form:

$$u^{(i)}(x) = f(x) + \int_0^x k(x-t)F\big(u(t)\big)dt. \tag{1.1}$$

We consider the kernel $k(x,t)$ as a difference kernel the depends on the differences $x-t$, such as e^{x-t}, $\cosh(x-t)$, and $\sinh(x-t)$.

This article is organized as follows: a brief introduction to our proposed method which depend on the modified Laplace Adomian decomposition method and the accelerated Adomian polynomial (El-Kalla polynomials). In section 3, the application of this method and numerical results are considered for the integro-differential equations by Laplace Adomian Decomposition Method (LADM) and making comparison tables. In section 4, ends this paper with the conclusions of our results.

2 The Modified Laplace Adomian Decomposition Method

In this section, we present steps of the proposed technique in solving non-linear integro-differential equations, which the approximated solution contains Adomian polynomial or El-Kalla polynomial.

$$u^{(i)}(x) = f(x) + \int_0^x k(x-t)F\big(u(t)\big)dt. \tag{2.1}$$

We recall that the Laplace transforms of the derivatives of $u(x)$ are defined by

$$\mathcal{L}\{u^{(i)}(x)\} = s^n\mathcal{L}\{u^n(x)\} - s^{n-1}u(0) - s^{n-2}u'(0) - \cdots - u^{(n-1)}(0). \tag{2.2}$$

This gives,

(I) $\mathcal{L}\{u'(x)\} = s\mathcal{L}\{u(x)\} - u(0) = sU(s) - u(0)$

(II) $\mathcal{L}\{u''(x)\} = s^2\mathcal{L}\{u(x)\} - su(0) - u'(0) = s^2U(s) - su(0) - u'(0).$

(III) $\mathcal{L}\{u'''(x)\} = s^3\mathcal{L}\{u(x)\} - s^2u(0) - su'(0) - u(0)$

$$= s^3U(s) - s^2u(0) - su'(0) - u(0)$$

and so on for derivatives of higher order, where $U(s) = \mathcal{L}\{u(x)\}$. Applying the Laplace transform to both sides of (2.1) gives

$$s^n\mathcal{L}\{u^n(x)\} - s^{n-1}u(0) - s^{n-2}u'(0) - \cdots - u^{n-1}(0) = \mathcal{L}\{f(x)\} + \mathcal{L}\big\{k(\mathrm{x}-\mathrm{t}) \otimes \mathrm{F}\big(\mathrm{u}(\mathrm{t})\big)\big\} \tag{2.3}$$

The Laplace of convolution term $k(x-t) \otimes F\big(u(t)\big)$ can be written as product of terms so,

$$s^n\mathcal{L}\{u^n(x)\} - s^{n-1}u(0) - s^{n-2}u'(0) - \cdots - u^{n-1}(0) = \mathcal{L}\{f(x)\} + \mathcal{L}\{k(x-t)\}\mathcal{L}\big\{F\big(u(t)\big)\big\} \tag{2.4}$$

This can be reduced to

$$\mathcal{L}\{u(x)\} = \frac{1}{s}u(0) + \frac{1}{s^2}u'(0) + \cdots + \frac{1}{s^n}u^{n-1}(0) + \frac{1}{s^n}\mathcal{L}\{f(x)\} \tag{2.5}$$

$$+\frac{1}{s^n}\mathcal{L}\{k(x-t)\}\mathcal{L}\{f(u))\}.$$

The Adomian decomposition method and the Adomian polynomials can be used to handle (2.5) and to address the non-linear term $F\big(u(x)\big)$. We first represent the linear term $u(x)$ at the left side by an infinite series of components given by

$$u(x) = \sum_{n=0}^{\infty} u_n\,(x). \tag{2.6}$$

where the components $u_n(x),\ n \geq 0$ will be recursively determined. However, the non-linear term $F\big(u(x)\big)$ at the right side of (2.5) will be represented by an infinite series of the Adomian polynomials A_n or Accelerated polynomials (El-Kalla polynomials) $\bar{A}_n$.

$$F\big(u(x)\big) = \sum_{n=0}^{\infty} A_n(x), \tag{2.7}$$

i. **Adomian Polynomials A_n**

$$A_n = \left(\frac{1}{n!}\right)\left(\frac{d^n}{d\lambda^n}\right)\left[F\left(\sum_{n=0}^{\infty}\lambda^i u_i\right)\right]_{\lambda=0}, n = 0,1,2,\ldots \tag{2.8}$$

If, $F(u(x)) = u^2(x)$, the Adomian polynomials are:

$$A_0 = u_0^2,$$

$$A_1 = 2u_0u_1,$$

$$A_2 = 2u_0u_2 + u_1^2,$$

$$A_3 = 2u_0u_3 + 2u_1u_2,$$

And $F\big(u(x)\big) = u^3(x)$, the Adomian polynomials are:

$$A_0 = u_0^3,$$

$$A_1 = 3u_0^2u_1,$$

$$A_2 = 3u_0^2u_2 + 3u_0u_1^2,$$

$$A_3 = 6u_0u_1u_2 + u_1^3 + 3u_0^2u_3.$$

ii. Accelerated Polynomials $\bar{A}_n$ formula

$$\bar{\mathrm{A}}_n = F(s_n) - \sum_{\mathrm{i}=0}^{n-1} \bar{\mathrm{A}}_n\,, \quad n = 0,1,2,\ldots \tag{2.9}$$

Where $\bar{A}_n$, are El-Kalla polynomial, $\bar{A}_0$, $\bar{A}_1$, $\bar{A}_2, \ldots$ and $F(s_n)$ is making a substitution of the summation of the solutions in the term of the non-linearity n-times. For instance, if the non-linear function is $F(u(x)) = u^2(x)$, the Accelerated polynomials $\bar{A}_n$ are:

$$\bar{A}_0 = u_0^2,$$

$$\bar{A}_1 = 2u_0u_1 + u_1^2,$$

$$\bar{A}_2 = 2u_0u_2 + 2u_1u_2 + u_2^2,$$

$$\bar{A}_3 = 2u_0u_3 + 2u_1u_3 + 2u_2u_3 + u_3^2,$$

and $F(u(x)) = u^3(x)$, the Accelerated polynomials $\bar{A}_n$:

$$\bar{A}_0 = u_0^3,$$

$$\bar{A}_1 = 3u_0^2u_1 + 3u_0u_1^2 + u_1^3,$$

$$\bar{A}_2 = 3u_0^2u_2 + 6u_0u_1u_2 + 3u_1^2u_2 + 3u_0u_2^2 + 3u_1u_2^2 + u_2^3,$$

$$\begin{aligned}\bar{A}_3 = {} & 3u_0^2u_3 + 6u_0u_1u_3 + 3u_1^2u_3 + 6u_0u_2u_3 + 6u_1u_2u_3 + 3u_2^2u_3 + 3u_0u_3^2 \\ & + 3u_1u_3^2 + 3u_2u_3^2 + u_3^2.\end{aligned}$$

Where $A_n,\ n \geq 0$ can be obtained for all forms of non-linearity. Substituting (2.6) and (2.7) into (2.5) leads to

$$\begin{aligned}\mathcal{L}\left\{\sum_{n=0}^{\infty} u_n(x)\right\} & \\ & = \frac{1}{s}u(0) + \frac{1}{s^2}u'(0) + \cdots + \frac{1}{s^n}u^{n-1}(0) + \frac{1}{s^n}\mathcal{L}\{f(x)\} \\ & + \frac{1}{s^n}\mathcal{L}\{k(x-t)\}\,\mathcal{L}\left\{\sum_{n=0}^{\infty} A_n(x)\right\}\end{aligned} \tag{2.10}$$

where,

$$\mathcal{L}\{u_0(x)\} = \frac{1}{s}u(0) + \frac{1}{s^2}u'(0) + \cdots + \frac{1}{s^n}u^{n-1}(0) + \frac{1}{s^n}\mathcal{L}\{f(x)\} \qquad (2.11)$$

$$\mathcal{L}\{u_{n+1}(x)\} = \frac{1}{s^n}\mathcal{L}\{k(x-t)\}\mathcal{L}\left\{\sum_{n=0}^{\infty} A_n(x)\right\}, \qquad n \geq 0. \qquad (2.12)$$

3 Numerical Results

In this section, three examples of non-linear integro-differential equations are presented to confirm the accuracy of the proposed method. The analytic solution for each problem is calculated. The comparison between the results of Laplace Adomian modifications and our proposed method will be introduced in this section.

Example 3.1 Consider the following non-linear Volterra integro-differential equation of the second [12] [16] [4]

$$u'(x) = -1 + \int_0^x u^2(t)dt\,, \;\; u(0) = 0. \qquad (3.1)$$

Applying the Laplace transform on (3.1) and by using the initial condition, we have:

$$sU(s) = -\frac{1}{s} + \mathcal{L}\left\{\int_0^x u^2(t)dt\right\}, \text{or}$$

$$U(s) = -\frac{1}{s^2} + \frac{1}{s}\mathcal{L}\left\{\int_0^x u^2(t)dt\right\} = -\frac{1}{s^2} + \frac{1}{s^2}\mathcal{L}\{u^2(t)\}.$$

Applying the inverse Laplace transform, we get

$$u(x) = -x + \mathcal{L}^{-1}\left\{\frac{1}{s}\mathcal{L}\left(\int_0^x u^2(t)dt\right)\right\}.$$

We decompose the solution can be expressed as an infinite series (2.6) and by using the recursive relation (2.12), the solution is:

i. Using Traditional Polynomials A_n

$u_0(x) = -x,$

$$u_1(x) = \frac{x^4}{12},$$

$$u_2(x) = -\frac{x^7}{252},$$

$$\vdots$$

$$u(x) = -x + \frac{x^4}{12} - \frac{x^7}{252} + \cdots.$$

ii. Using Accelerated Polynomials $\bar{A}_n$

$$u_0(x) = -x,$$

$$u_1(x) = \frac{x^4}{12},$$

$$u_2(x) = -\frac{x^7}{252} + \frac{x^{10}}{12960},$$

$$u(x) = -x + \frac{x^4}{12} - \frac{x^7}{252} + \frac{x^{10}}{12960} + \cdots.$$

Table 3.1 illustrates the comparison between solutions of Wavelet-Galerkin method (WGM), variation iteration method (VIM), Adomian decomposition method (ADM), and Laplace Adomian decomposition method with traditional polynomial (LADM) and accelerated polynomial (ALADM).

Table 3.1: shows a comparison between WGM, VIM, ADM, LADM and ALADM of Example 3.1

x	**WGM** [16]	**VIM** [12]	**ADM** [16]	**LADM** [12]	**Accelerated. LADM**
0.0000	0	0	0	0	0
0.0938	-0.0937	-0.9379355	-0.0937935	-0.09379	-0.09379
0.2188	-0.2186	-0.5186091	-0.2186090	-0.21861	-0.21861
0.3125	-0.3117	-0.317065	-0.3117060	-0.31171	-0.31171
0.4062	-0.4040	-0.4039385	-0.4039390	-0.40394	-0.40394
0.5000	-0.4948	-0.4948225	-0.4948230	-0.49482	-0.49482
0.6250	-0.6124	-0.6124306	-0.6124310	-0.61243	-0.61243
0.7188	-0.6969	-0.6969414	-0.669410	-0.69694	-0.69694
0.8125	-0.7771	-0.7770901	-0.7770900	-0.77709	-0.77709
0.9062	-0.8520	-0.8519340	-0.8519340	-0.85193	-0.85193
1.0000	-0.9205	-0.904747	-0.9204760	-0.92047	-0.92047

Example 3.2 Consider the following non-linear Volterra integro-differential equation of the second kind [17]

$$u'(x) = \frac{9}{4} - \frac{5}{2}x - \frac{1}{2}x^2 - 3e^{-x} - \frac{1}{4}\,e^{-2x} + \int_0^x (x-t)\,u^2(t)dt, u(0) = 2, \quad (3.2)$$

and the exact solution is $u(x) = 1 + e^{-x}$.

Taking Laplace transform of both sides of equation (3.2) gives:

$$\mathcal{L}\{u'(x)\} = \mathcal{L}\left\{\frac{9}{4} - \frac{5}{2}x - \frac{1}{2}x^2 - 3e^{-x} - \frac{1}{4}e^{-2x}\right\} + \mathcal{L}\{(x-t) \otimes u^2(x)\},$$

so that,

$$sU(s) - u(0) = \frac{9}{4s} - \frac{5}{2s^2} - \frac{1}{s^3} - \frac{3}{s+1} - \frac{1}{4(s+2)} + \frac{1}{s^2}\mathcal{L}\{u^2(x)\},$$

or equivalently,

$$U(s) = \frac{2}{s} + \frac{9}{4s^2} - \frac{5}{2s^3} - \frac{1}{s^4} - \frac{3}{s(s+1)} - \frac{1}{4s(s+2)} + \frac{1}{s^3}\mathcal{L}\{u^2(x)\}.$$

Substituting the series assumption for $U(s)$ and the Adomian polynomials for $u^2(x)$ as given above in (2.6) and equations (2.8) and (2.9) respectively, and using the recursive relation (2.12), we obtain

$$U_0(s) = \frac{2}{s} + \frac{9}{4s^2} - \frac{5}{2s^3} - \frac{1}{s^4} - \frac{3}{s(s+1)} - \frac{1}{4s(s+2)}$$

$$\mathcal{L}\{u_{k+1}(x)\} = \frac{1}{s^3}\mathcal{L}\{A_k(x)\}, \quad k \geq 0.$$

where, A_k are the Adomian polynomials for $u^2(t)$, take Laplace inverse $\mathcal{L}^{-1}$

i. Using Traditional Polynomials A_n

$$u_0(x) = -\frac{9}{8} + \frac{e^{-x}}{8} + 3e^{-x} + \frac{9x}{4} - \frac{5x^2}{4} - \frac{x^3}{6}$$

And Taylor expansion is:

$$u_0(x) = 2 - x + \frac{x^2}{2} - \frac{5x^3}{6} + \frac{5x^4}{24} - \frac{7x^5}{120} + \cdots, \text{and so on,}$$

$$u(x)=\sum_{n=0}^{\infty}u_n=u_0+u_1+u_2+\cdots$$

$$u(x)=2-x+\frac{x^2}{2}-\frac{x^3}{6}+\frac{x^4}{24}-\frac{x^5}{120}-\frac{x^6}{72}+\frac{x^7}{280}-\frac{17x^8}{20160}-\frac{181x^9}{181440}+\frac{409x^{10}}{907200}-\frac{137x^{11}}{798336}+\frac{43x^{12}}{534600}-\frac{3241x^{13}}{111196800}+\frac{103507x^{14}}{10897286400}-\frac{917219x^{15}}{326918592000}+\frac{214783x^{16}}{290594304000}-\frac{54101x^{17}}{302455296000}+\frac{350431x^{18}}{10888390656000}-\frac{2723x^{19}}{574665062400}+\frac{49x^{20}}{112679424000}-\frac{49x^{21}}{1690191360000}+\cdots$$

That converges to the exact solution

$$u(x)=1+e^{-x}$$

ii. Using Accelerated Polynomials $\bar{A}_n$

$u_0(x)=2-x+\frac{x^2}{2}-\frac{5x^3}{6}+\frac{5x^4}{24}-\frac{7x^5}{120}+\cdots$, and so on

Then, the solution is

$$u(x)=2-x+\frac{x^2}{2}-\frac{x^3}{6}+\frac{x^4}{24}-\frac{x^5}{120}-\frac{x^6}{72}+\frac{x^7}{280}-\frac{17x^8}{20160}-\frac{x^9}{8640}+\frac{43x^{10}}{302400}-\frac{169x^{11}}{2217600}+\frac{67x^{12}}{2138400}-\frac{743x^{13}}{70761600}+\frac{34339x^{14}}{10897286400}-\frac{105067x^{15}}{326918592000}-\frac{1423x^{16}}{14529715200}+\frac{1852237x^{17}}{22230464256000}-\frac{6905749x^{18}}{145508493312000}+\frac{1358767x^{19}}{72754246656000}-\frac{55443137x^{20}}{8944492677120000}+\frac{331457317x^{21}}{187834346219520000}-\frac{97781x^{22}}{217275125760000}+\frac{37204393x^{23}}{357308944312320000}-\frac{20177x^{24}}{972269236224000}+\frac{344819x^{25}}{97226923622400000}-\frac{262969x^{26}}{549543481344000000}$$

$$+\frac{8477x^{27}}{164863044403200000}-\frac{49x^{28}}{131890435522560000}$$
$$+\frac{343x^{29}}{19124113150771200000}+\cdots$$

Table 3.2 shows absolute error between (LADM) solution and ALADM solution and exact solution. Figure 3.1 shows the exact solution, LADM, and ALADM and Figure 3.2 shows the absolute error of LADM, and ALADM.

Table 3.2: shows a comparison between LADM and ALADM of Example 3.2

x	$u_{\mathrm{Ex}}(x)$	$u_{\mathrm{LADM}}(x)$	$u_{\mathrm{ALDM}}(x)$	$\lvert u_{\mathrm{Ex}}-u_{\mathrm{LADM}}\rvert$	$\lvert u_{\mathrm{Ex}}-u_{\mathrm{ALDM}}\rvert$
0.0	2	2	2	0	0
0.1	1.904837	1.904837	1.904837	1.49E-08	1.49E-08
0.2	1.818731	1.81873	1.81873	9.32E-07	9.32E-07
0.3	1.740818	1.740808	1.740808	1.04E-05	1.04E-05
0.4	1.67032	1.670263	1.670263	5.72E-05	5.7E-05
0.5	1.606531	1.606316	1.606318	0.000214	0.000213
0.6	1.548812	1.548182	1.548189	0.00063	0.000622
0.7	1.496585	1.495018	1.495047	0.001567	0.001539
0.8	1.449329	1.445873	1.445964	0.003456	0.003365
0.9	1.40657	1.399615	1.399868	0.006955	0.006701
1.0	1.367879	1.354846	1.35548	0.013033	0.0124

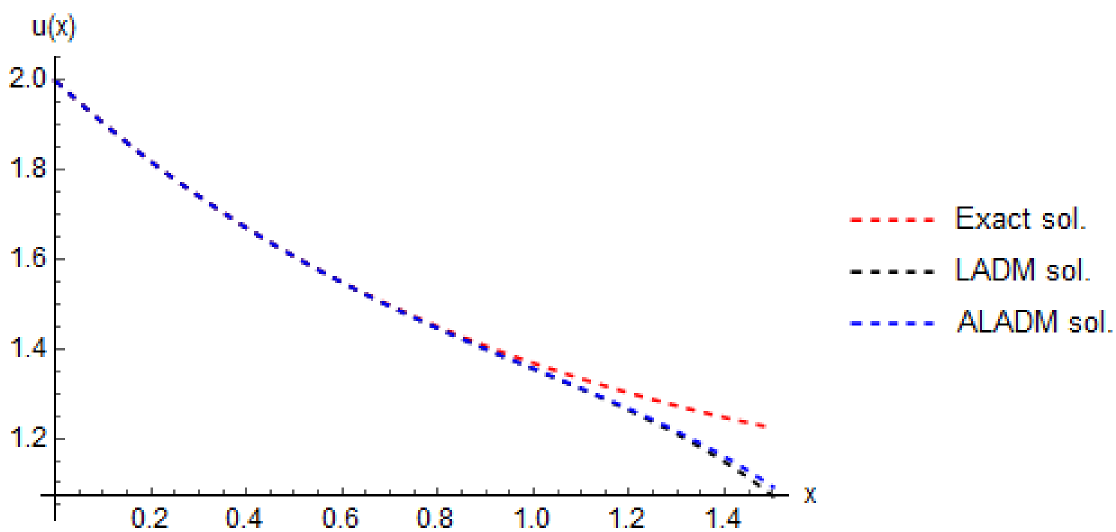

Figure 3.1: shows the comparison between exact and Adomian, and Accelerated solutions of Example 3.3

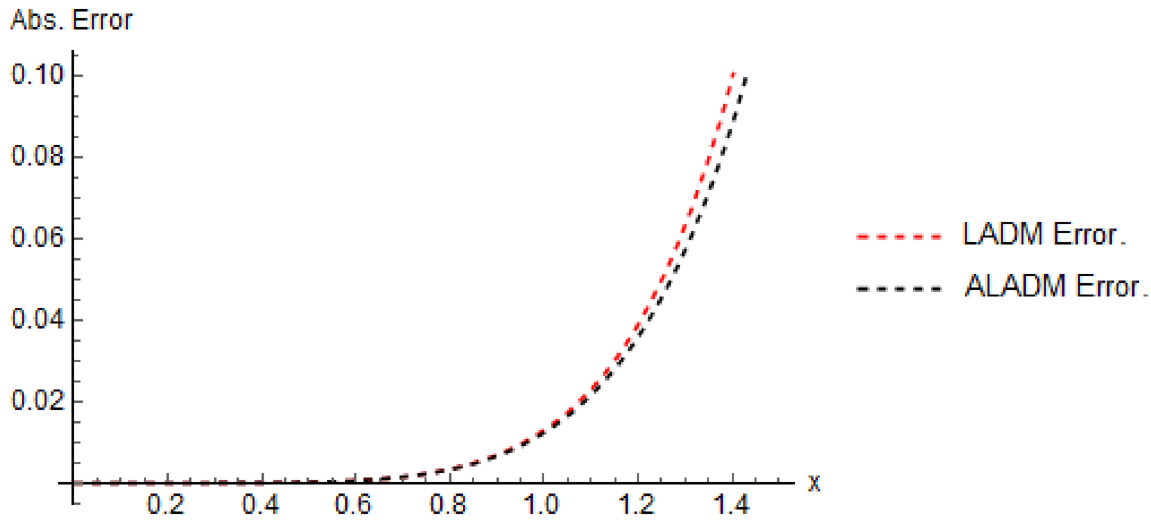

Figure 3.2: shows the absolute error of LADM and ALADM of Example 3.3 .

Example 3.3 Consider the non-linear integro-differential equation of the second kind [2]

$$u'(x) = \frac{3}{2}e^{x} - \frac{1}{2}e^{3x} + \int_{0}^{x} e^{x-t}\, u^{3}(t)dt, \quad u(0) = 1. \qquad (3.3)$$

Notice that the kernel $k(x-t) = e^{x-t}$. Taking Laplace transform of both sides of above equation (3.3).

$$\mathcal{L}(u'(x)) = \mathcal{L}\left(\frac{3}{2}e^{x} - \frac{1}{2}e^{3x}\right) + \mathcal{L}(e^{x-t} + u^{3}(x)) \,,$$

so that,

$$sU(s) - u(0) = \frac{3}{2(s-1)} - \frac{1}{2(s-3)} + \frac{1}{s-1}\mathcal{L}\big(u^{3}(x)\big),$$

or equivalently

$$U(s) = \frac{1}{s} + \frac{3}{2s(s-1)} - \frac{1}{2s(s-3)} + \frac{1}{s(s-1)}\mathcal{L}\big(u^{3}(x)\big) \,,$$

where $U(s)=\mathcal{L}\{u(x)\}$. Substituting the series assumption for $U(s)$ and the Adomian and El-Kalla polynomials for $u^3(x)$ as given above in (2.8) and (2.9) respectively, and using the recursive relation (2.12), we obtain:

$$U_0(s)=\frac{1}{s}+\frac{3}{2s(s-1)}-\frac{1}{2s(s-3)},$$

$$\mathcal{L}\{u_{k+1}(x)\}=\frac{1}{s(s-1)}\mathcal{L}\{A_k(x)\},\ \ k\geq 0,$$

taking the inverse Laplace inverse $\mathcal{L}^{-1}$for $U_0(s)$ and using the recurrence relations gives:

i. Using Traditional Polynomials

$$u_0(x)=1+x-\frac{x^3}{2}-\frac{x^4}{2}-\frac{13x^5}{40}-\frac{x^6}{6}+\cdots$$

$$u_1(x)=\frac{x^2}{2}+\frac{2x^3}{3}+\frac{5x^4}{12}+\frac{7x^5}{120}-\frac{101x^6}{720}+\cdots$$

$$u_2(x)=\frac{x^4}{8}+\frac{11x^5}{40}+\frac{13x^6}{48}+\cdots$$

$\vdots$

$$u(x)=1+x+\frac{x^2}{2}+\frac{x^3}{6}+\frac{x^4}{24}+\frac{x^5}{120}+\cdots.$$

We can rewrite as:

$$u(x)=1+x+\frac{1}{2!}x^2+\frac{1}{3!}x^3+\frac{1}{4!}x^4+\cdots.$$

That converges to the exact solution

$$u(x)=e^x$$

ii. Using Accelerated Polynomials

$$u_0(x)=1+x-\frac{x^3}{2}-\frac{x^4}{2}-\frac{13x^5}{40}-\frac{x^6}{6}+\cdots$$

$$u_1(x)=\frac{x^2}{2}+\frac{2x^3}{3}+\frac{5x^4}{12}+\frac{7x^5}{120}-\frac{101x^6}{720}+\cdots$$

$$u_2(x) = \frac{x^4}{8} + \frac{11x^5}{40} + \frac{71x^6}{240} + \cdots$$

$\vdots$

$$u(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} - \frac{x^6}{90} + \cdots.$$

We can rewrite as:

$$u(x) = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \cdots.$$

That converges to the exact solution

$$u(x) = e^x.$$

Table 3.3: shows a comparison between LADM and ALDM solution of Example 3.3 . Figure 3.3 shows the exact solution, LADM, and ALADM and Figure 3.4 shows the absolute error of LADM, and ALADM.

Table 3.3: shows a comparison between LADM and ALDM solution of Example 3.3

x	$u_{\mathrm{Ex}}(x)$	$u_{\mathrm{LADM}}(x)$	$u_{\mathrm{ALDM}}(x)$	$\lvert u_{\mathrm{Ex}} - u_{\mathrm{LADM}}\rvert$	$\lvert u_{\mathrm{Ex}} - u_{\mathrm{ALDM}}\rvert$
0.0	1	1	1	0	0
0.1	1.105171	1.105171	1.105171	3.75E-08	1.25E-08
0.2	1.221403	1.2214	1.221402	2.4E-06	8.03E-07
0.3	1.349859	1.349831	1.34985	2.74E-05	9.16E-06
0.4	1.491825	1.491671	1.491773	0.000154	5.15E-05
0.5	1.648721	1.648134	1.648524	0.000588	0.000197
0.6	1.822119	1.820363	1.82153	0.001756	0.000589
0.7	2.013753	2.009323	2.012264	0.00443	0.001489
0.8	2.225541	2.215664	2.222218	0.009877	0.003323
0.9	2.459603	2.439567	2.452853	0.020036	0.00675
1.0	2.718282	2.680556	2.705556	0.037726	0.012726

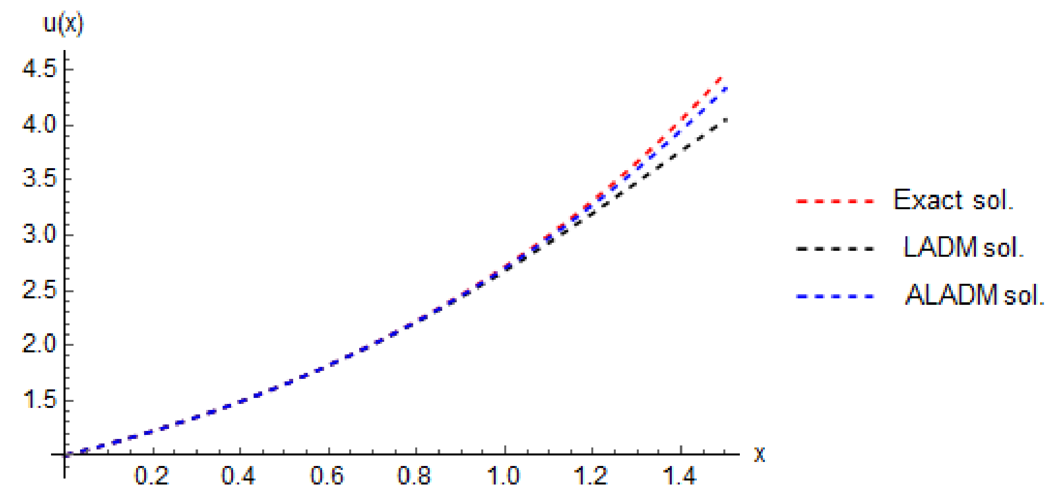

Figure 3.3: shows the comparison between exact and Adomian, and Accelerated solutions of Example 3.3

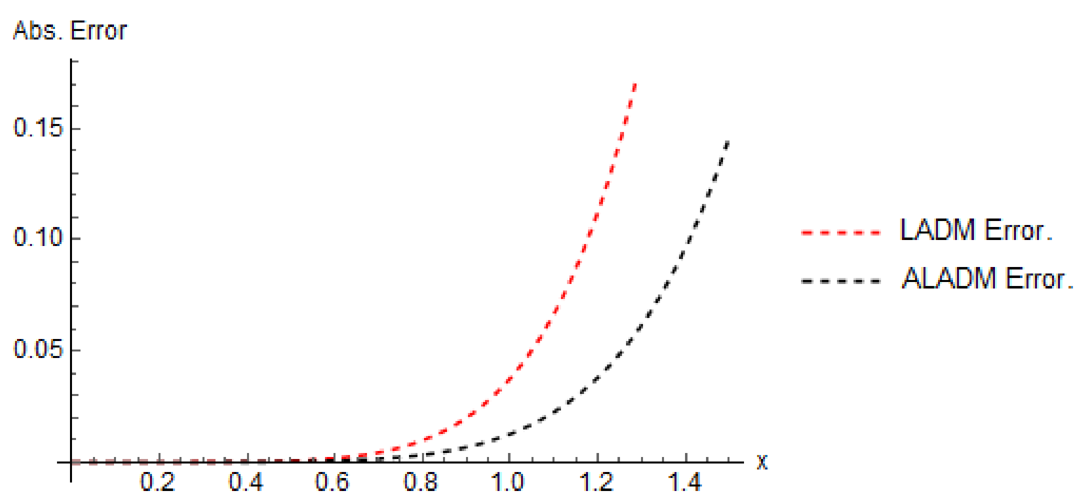

Figure 3.4: shows the absolute error of LADM and ALADM of Example 3.3 .

4 Conclusion

In this paper, we approximate a semi analytic solution of the non-linear Volterra integro-differential equation by using Laplace Adomian decomposition method and using two approaches of non-linear terms. We demonstrated that the solution of El-Kalla polynomials $\bar{A}_n$ is quite efficient and accelerated convergent than Adomian polynomials A_n.

References

[1] N. Bildik and M. Inc, "A comparison between Adomian decomposition and Tau methods," in *Abstract and Applied Analysis*, 2013, vol. 2013.
[2] A.-M. Wazwaz, "The combined Laplace transform–Adomian decomposition method for handling nonlinear Volterra integro–differential equations," *Appl. Math. Comput.*, vol. 216, no. 4, pp. 1304–1309, 2010.
[3] M. T. Rashed, "Lagrange interpolation to compute the numerical solutions of differential, integral and integro-differential equations," *Appl. Math. Comput.*, vol. 151, no. 3, pp. 869–878, 2004.
[4] A. Avudainayagam and C. Vani, "Wavelet–Galerkin method for integro–differential equations," *Appl. Numer. Math.*, vol. 32, no. 3, pp. 247–254, 2000.
[5] A.-M. Wazwaz, "A comparison study between the modified decomposition method and the traditional methods for solving nonlinear integral equations," *Appl. Math. Comput.*, vol. 181, no. 2, pp. 1703–1712, 2006.
[6] S. A. Khuri, "A Laplace decomposition algorithm applied to a class of nonlinear differential equations," *J. Appl. Math.*, vol. 1, no. 4, pp. 141–155, 2001.
[7] M. Khan, M. Hussain, H. Jafari, and Y. Khan, "Application of Laplace decomposition method to solve nonlinear coupled partial differential equations," *World Appl. Sci. J.*, vol. 9, no. 1, pp. 13–19, 2010.
[8] N. S. Elgazery, "Numerical solution for the Falkner–Skan equation," *Chaos Solitons Fractals*, vol. 35, no. 4, pp. 738–746, 2008.
[9] S. Islam, Y. Khan, N. Faraz, and F. Austin, "Numerical solution of logistic differential equations by using the Laplace decomposition method," *World Appl. Sci. J.*, vol. 8, no. 9, pp. 1100–1105, 2010.
[10] C. Yang and J. Hou, "An Approximate Solution of Nonlinear Fractional Differential Equation by Laplace Transform and Adomian Polynomials ⋆," *J. Inf.*, p. 11, 2013.
[11] D. J. Evans and K. R. Raslan, "The Adomian decomposition method for solving delay differential equation," *Int. J. Comput. Math.*, vol. 82, no. 1, pp. 49–54, 2005.
[12] J. M. Heris, "Solving the Integro-Differential Equations Using the Modified Laplace Adomian Decomposition Method," *J. Math. Ext.*, vol. 6, 2012.
[13] M. A. Mohamed and M. S. Torky, "Numerical solution of nonlinear system of partial differential equations by the Laplace decomposition method and the Pade approximation," *Am. J. Comput. Math.*, vol. 3, no. 3, p. 175, 2013.
[14] J. B. Yindoula, P. Youssouf, G. Bissanga, F. Bassono, and B. Some, "Application of the Adomian decomposition method and Laplace transform method to solving the convection diffusion-dissipation equation," *Int. J. Appl. Math. Res.*, vol. 3, no. 1, p. 30, 2014.
[15] A. A. Hamoud and K. P. Ghadle, "The combined modified Laplace with Adomian decomposition method for solving the nonlinear Volterra-Fredholm integro differential equations," *J. Korean Soc. Ind. Appl. Math.*, vol. 21, no. 1, pp. 17–28, 2017.
[16] S. M. El-Sayed and M. R. Abdel-Aziz, "A comparison of Adomian's decomposition method and wavelet-Galerkin method for solving integro-differential equations," *Appl. Math. Comput.*, vol. 136, no. 1, pp. 151–159, 2003.
[17] A.-M. Wazwaz, *Linear and nonlinear integral equations*, vol. 639. Springer, 2011.