



Effect of the variability of consolidation parameters for clay improved by wick drains using Monte Carlo simulations

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Abstract. This paper explores the quantification of uncertainties in predicting the settlement of embankments constructed on soft soil deposits enhanced with preloading and prefabricated vertical drains (PVDs). Settlement values often vary significantly due to uncertainties, particularly those associated with overconsolidation, compressibility properties, and the stratification of clay layers. Accurately identifying these uncertainties poses a considerable challenge because of the typically limited and variable data available. To address these uncertainties, reliability analysis provides a robust approach. Instead of relying on a single deterministic settlement value, reliability analysis estimates the probability distribution of consolidation settlement, offering a deeper insight into the outcomes. Stochastic processes, such as Monte Carlo simulations (MC), are commonly employed for this purpose. Monte Carlo simulations involve generating random variables within established models to simulate a range of possible results. This method effectively assesses the impact of uncertainty and risk on settlement predictions, enabling greater accuracy in the analysis.

In this study, settlement predictions were statistically computed based on results obtained from PLAXIS 2D finite element modeling. Additionally, the study examines various factors influencing the reliability analysis of clay consolidation settlement. These factors include the number of simulations performed, the level of uncertainty in soil consolidation parameters, and the stresses induced by structural loads.

Keywords: Prefabricated vertical drains, Monitoring, Stochastic analysis.

1 Introduction

Soft clay soils are widespread across many regions globally and pose significant challenges due to their high compressibility and low bearing capacity [7]. Various soil improvement techniques have been

explored to address these challenges [2]. The primary objective of such methods is to mitigate deformations in structures built on soft clay. The preloading method, often combined with vertical drains, is commonly used to expedite the consolidation process. This combination is essential because the low permeability of soft clay significantly extends the time required to achieve the desired degree of consolidation when relying solely on preloading.

Vertical drains can be installed using different materials, including gravel, sand, or stones, as well as prefabricated synthetic options known as prefabricated vertical drains (PVDs) [5]. The combination of preloading with PVDs has recently emerged as one of the most effective and cost-efficient solutions for improving soft clay soils [8].

Both numerical and analytical methods can yield realistic results; however, the evaluation of soil parameters plays a critical role in determining the accuracy of these analyses [12]. Due to the variability and randomness of soil profiles, deterministic calculations often fall short in accurately predicting settlement values for structures [9]. Consequently, it becomes essential to quantify uncertainties in soil parameters, as these uncertainties significantly influence calculated settlements. A clear understanding of these concepts enables effective methods to quantify and address these uncertainties [3]. If the predicted settlement values deviate considerably from the calculated ones, this can facilitate decision-making for clients, allowing them to assess risks and the associated economic costs of corrective actions. By adopting appropriate measures, risks can be mitigated, and costs minimized while maximizing benefits [6].

The terms "risk," "uncertainty," and "safety" are widely recognized in geotechnical engineering analysis and design. Despite extensive site investigation and field monitoring, particularly in large-scale projects, which aim to produce the most reliable data for geotechnical analysis, it is crucial to evaluate and quantify the uncertainties involved [11]. Reliability analysis offers a framework to incorporate the effects of these uncertainties into the results. This analysis relies on statistical and probabilistic principles, which are valuable for determining the mean value (most probable value) and the range of values for input soil parameters, thereby influencing the output results [13]. In essence, these methods allow for the quantification of uncertainties, enabling a more rational and systematic consideration of variable randomness.

This study examines three key factors influencing the uncertainty of consolidation settlement. The first is the number of simulations used in the analysis. Choosing an appropriate number of simulations is essential for achieving accurate reliability results without conducting unnecessary iterations, which can be time-intensive. In this research, different simulation counts will be tested to determine the optimal balance.

The second factor is the uncertainty of soil parameters, particularly those related to consolidation, which significantly impact the compressibility of clay. This uncertainty is quantified using the coefficient of variation (CoV), with various CoV values representing high, medium, and low uncertainty levels.

The third factor is the final stresses anticipated from structural loads. These stresses have a direct impact on the reliability analysis results for settlement. Each of these factors is carefully considered in the study to provide a comprehensive assessment of uncertainty in consolidation settlement predictions.

2 Soil Formations

The soil profile for the condition under study predominantly consists of an upper soft clay layer with a thickness of 28 meters, followed by a sand layer approximately 15 meters thick, then a hard clay layer, and finally a deep sand layer. The upper clay layer is divided into seven sublayers to account for the variation in soil parameters with depth.

Table 1 provides a summary of the idealized soil profile along with the mean values of selected soil parameters. These include unit weight (γ), overconsolidation ratio (OCR), drained cohesion (C'),

drained angle of internal friction (ϕ'), vertical permeability (K_v), and horizontal permeability (K_h). Additionally, the compressibility parameters are also detailed in table 2, including the compression index (C_c), re-compression index (C_r), and initial void ratio (e_o). These parameters form the basis for analyzing the behavior of the soil profile under the given conditions.

Table 1. Idealized soil profile and parameters

Layer	Top Level	Bottom Level	γ	OCR	C' (KPa)	ϕ'	K_v (cm/s)	K_h (cm/s)
			(KN/m ³)			(Degrees)		
Clay_1	1.9	0	13.9	2.6	3	24.5	2.3E-08	7E-08
Clay_2	0	-3	14	6.2	3	24.5	2.3E-08	7E-08
Clay_3	-3	-6	13.7	1.9	3	24.5	2.3E-08	7E-08
Clay_4	-6	-10	15.6	3	3	24.5	2.3E-08	7E-08
Clay_5	-10	-14	15.6	1.9	3	20.5	2.3E-08	7E-08
Clay_6	-14	-25.3	16.4	1.5	3.7	20.5	2.3E-08	7E-08
Clay_7	-25.3	-26	18.2	2.7	4.4	20.5	2.3E-08	7E-08
Sand	-26	-40.5	19	--	--	40	1.7	--
Lower Clay	-40.5	-41.8	19	5	20	21	2.3E-08	7E-08
Sand	-41.8	---	19	--	--	40	1.7	--

For the clay layers, table 2 below summarizes the clay layers compressibility parameters.

Table 2. Clay layers compressibility parameters

Compressibility parameter	Value
C_c	0.585
C_r	0.065
e_o	1.61

3 Construction stages

For the conditions under study, the following considerations were applied:

- The natural ground level is set at +1.90 meters.
- Vertical drains are installed in a square pattern with 1.50m x 1.50m spacing.
- The tip level of the vertical drains reaches -25.00 meters.
- The top level of the embankment is raised to +4.95 meters.
- The preloading duration is approximately 5.5 months.
- The embankment's top dimensions are 220m x 220m.

Figure 1 illustrates a section of the preloading embankment. A detailed construction sequence was adopted in the 2D finite element modeling to simulate the preparation of the platform, the installation of wick drains, the application of preloading, the removal of the preloading embankment, and the addition of the final structural loads. Table 3 provides a summary of the adopted construction sequence.

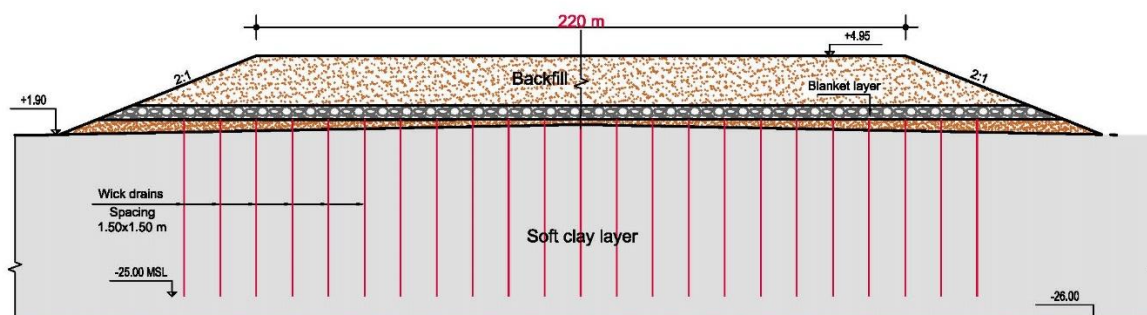


Fig. 1. Section of preloading embankment

Table 3: Adopted construction sequence in modelling

Stage No.	Description	Notes
1	Backfill of 0.2m to prepare platform at level +2.10	Surcharge load of 3.6 KPa
2	Install the wick drains	Period is 20 days
3	Backfill an additional 2.85m to level +4.95	Total surcharge is load 55 KPa during period 60 days
4	Perform the preloading	Consolidation for 165 days
5	Final consolidation settlement due to the preloading load	100% consolidation
6	Remove the preload, excavate & add replacement soil layer below foundations.	The foundation level assumed +0.70 and the replacement thickness assumed 0.5m. This stage starts from end of the stage 4
7	Add the building load	The building stress is assumed 20 KPa
8	Final consolidation settlement due to the structure loading	100% consolidation

4 Finite Element Analysis

The analyses were performed using 2D modeling in PLAXIS 2D software. Consolidation analysis was included at Stage 5 after preloading to evaluate the minimum pore pressure and estimate the final consolidation settlement. While this phase does not represent realistic project construction, it aids in estimating the consolidation settlement and the degree of consolidation achieved at the end of preloading.

5 Model Configuration

A plain strain simulation was utilized in the PLAXIS 2D analysis. The preloading embankment width was set at 110 meters, while the horizontal model boundary extended to 200 meters. This configuration ensures that the surcharge stresses from the preloading and final structures cover an area of approximately 220m x 220m. Figure 2 illustrates the PLAXIS 2D model configuration.

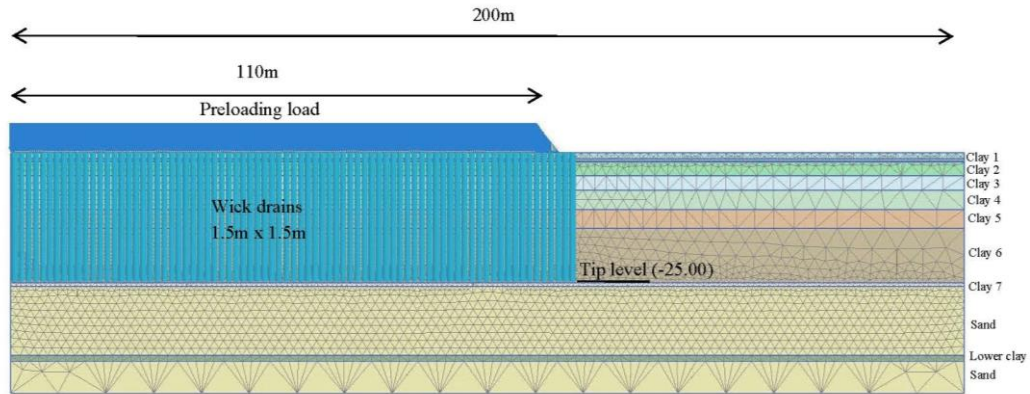


Fig. 2. PLAXIS 2D model configuration

6 Constitutive Models and Material Properties

The Soft Soil model [5] was applied to the clay layers, while the sand layers were analyzed using the Mohr-Coulomb model. For the soft clay, undrained analysis using Method A was implemented, employing drained parameters to represent undrained soil behavior.

The smear effect caused by wick drain installation in clay layers was addressed using the equivalent soil parameter method [10]. This method replaces the disturbed and undisturbed zones around the vertical drains with an equivalent soil cluster having identical parameters to the undisturbed soil, except for permeability.

The equivalent permeability (K_e) is determined using Lin's equation as follows:

$$K_e = \frac{K_h \ln\left(\frac{r_e}{r_w}\right)}{\ln\left(\frac{r_e}{r_s}\right) + \frac{K_h}{K_s} \ln\left(\frac{r_s}{r_w}\right)} \quad (1)$$

Where:

- r_w : Equivalent radius of the vertical drain
- r_e : Radius of influence zone
- r_s : Radius of smear zone
- K_e : Equivalent soil permeability
- K_h : Horizontal permeability of the undisturbed soil
- K_s : Permeability of the smear zone

Based on studies of soft Bangkok clay [4], the smear zone diameter ranges between two and three times the mandrel's cross-sectional area, with permeability between one-third and one-half of the undisturbed soil's permeability. Consequently, the equivalent permeability is approximately half of the undisturbed soil permeability.

Drains were modeled as line drains, while surface loads represented the platform and preloading backfill. A backfill density of 18 kN/m^3 was assumed for both the platform and preloading works.

7 Reliability Analysis Overview

The reliability analysis employs Monte Carlo (MC) simulations, a robust numerical technique for addressing stochastic problems by estimating the statistical behavior of random input data and corresponding output variables. The method is versatile, applicable to linear and non-linear problems, and utilizes a large number of simulations to establish a reliable response distribution.

7.1 Monte Carlo Simulation Method

MC simulations are implemented using Latin Hypercube Sampling (LHS) from the pyDOE Python library. LHS is a stratified sampling technique that divides the probability distribution into equal-sized intervals, ensuring a comprehensive representation of input distributions in the sampled data. This enhances the accuracy of the statistical analysis.

PLAXIS 2D scripting tools with Python coding can automate the reliability analysis, incorporating MC simulations to evaluate the stochastic behavior of the system. The number of simulations is a critical parameter, influencing the precision of results and must be carefully chosen.

7.2 Statistical Parameters

Key statistical measures for random variables include:

- Mean (μ): The arithmetic average of data points.
- Standard Deviation (σ): A measure of variability in the data (variable X distribution with n data points), is calculated as follows:

$$\sigma = \sqrt{\frac{\sum(X_i - \bar{X})^2}{n}} \quad (2)$$

- Coefficient of Variation (CoV): The ratio of the standard deviation to the mean, indicating the relative variability in the data, is calculated as follows:

$$CoV = \frac{\text{standard deviation}}{\text{mean value}} \quad (3)$$

For reliability analysis, statistical data should be derived from extensive site investigations to ensure accurate representation of soil parameter variability. Compressibility parameters for soft clay layers, which significantly influence consolidation settlement, are particularly important.

8 Results and Discussion

8.1 Effect of Number of Monte Carlo Simulations`

The reliability analysis examines three settlement cases, focusing on the influence of the number of simulations on accuracy:

1. Settlement Case 1:
 - Settlement from the start of backfilling (Stage 1) to the end of preloading (Stage 4).
 - Measured during construction using Deep Settlement Plates (DSP) placed at the bottom of the embankment.

Settlement Case 2:

- Settlement from the final backfill level (Stage 3) to the end of preloading (Stage 5).
- Measured during construction using Surface Settlement Plates (SSP) installed on top of the embankment.

2. Settlement Case 3:

- Total settlement from the start of backfilling (Stage 1) to final consolidation (Stage 5) using preloading load.
- Estimated using methods such as Asaoka [1] and Hyperbolic methods [14], based on recorded settlements during construction.

The key compressibility parameters used for the study include:

- OCR: Overconsolidation ratio.
- λ^* : Normalized compression index, is calculated as follows:

$$\lambda^* = \frac{Cc}{2.3(1+eo)} \quad (4)$$

- κ^* : Normalized re-compression index, is calculated as follows:

$$\kappa^* = \frac{2 Cr}{2.3(1+eo)} \quad (5)$$

Table 4 summarizes the statistical data for these parameters, demonstrating their variability and contribution to settlement behavior. The number of MC simulations directly impacts the precision of settlement predictions for each case (Figures 3-5).

This analysis underscores the importance of selecting an appropriate number of simulations and accurately characterizing input parameters through comprehensive site investigations for reliable consolidation settlement predictions.

Table 4. Statistical data for soil compressibility parameters

Layer	OCR			λ^*			κ^*		
	μ	σ	CoV	μ	σ	CoV	μ	σ	CoV
Clay_1	2.5	1.13	0.45	0.0965	0.0164	0.17	0.0217	0.00256	0.12
Clay_2	6.2	2.9	0.47						
Clay_3	1.9	1.35	0.71						
Clay_4	3	1.9	0.63						
Clay_5	1.9	0.5	0.26						
Clay_6	1.5	0.22	0.15						
Clay_7	2.7	1.9	0.7						

The relationship between settlement values and frequency percentages was analyzed for different numbers of simulations, ranging from 100 to 1000.

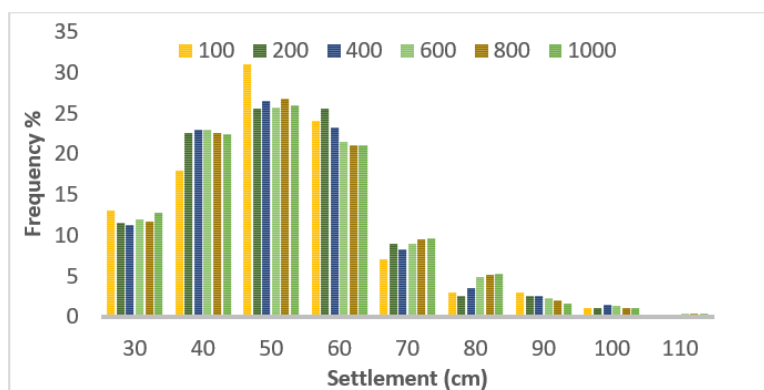


Fig. 3. Reliability analysis results using different simulation numbers – Settlement case 1

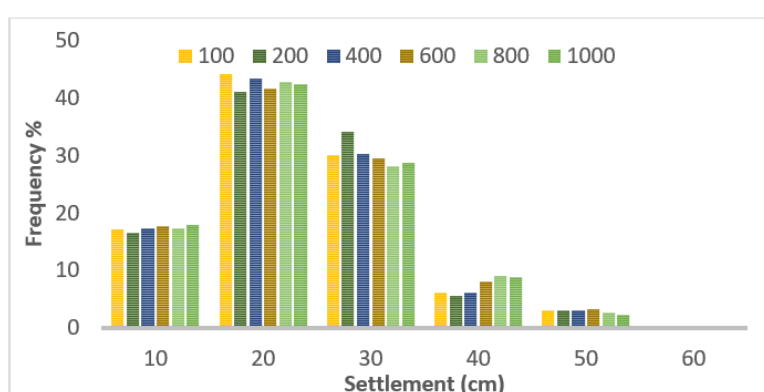


Fig. 4. Reliability analysis results using different simulation numbers – Settlement case 2

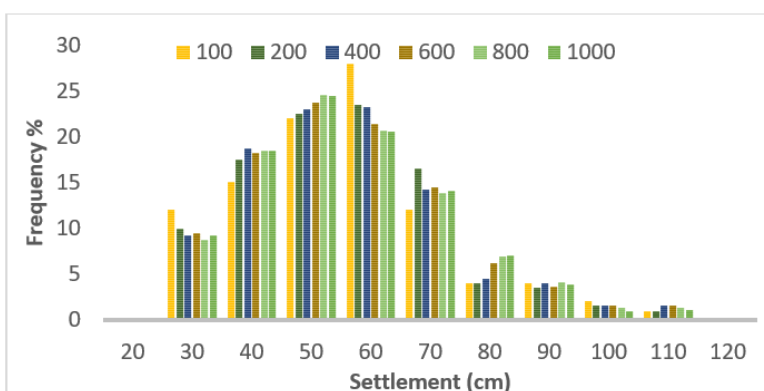


Fig. 5. Reliability analysis results using different simulation numbers – Settlement case 3

The results of 1000 simulations were used as the reference point, representing the highest accuracy in reliability analysis. Based on the data plotted in Figures 3, 4, and 5, the following observations were drawn regarding the error percentages associated with various numbers of simulations:

- For $N = 100$ simulations:
The average error percentage varies between 11% and 22.2%, indicating relatively low precision.

- For N = 200 simulations:

The average error percentage decreases to a range of 8% to 15.1%, reflecting moderate improvement.

- For N = 400 simulations:

A further reduction in error is observed, with values ranging between 8.4% and 10.4%, showing improved accuracy.

- For N = 600 simulations:

The error percentage significantly decreases to between 3.1% and 4.6%, approaching high precision.

- For N = 800 simulations:

Minimal error is achieved, ranging from 2.2% to 2.8%, closely aligning with the results of 1000 simulations.

8.2 Effect of Coefficient of Variation (CoV)

The Coefficient of Variation (CoV) reflects the degree of uncertainty in consolidation parameters for the upper clay layer. In this study, the values of CoV considered are 0.1, 0.2, and 0.3, representing low, medium, and high uncertainty, respectively. Key parameters analyzed include OCR, C_c , C_r , and e_o across the seven upper clay sublayers.

Four settlement cases were analyzed with the PLAXIS 2D model, using 100 iterations for each CoV value:

1. Case 1: From start of backfilling (Stage 1) to the end of preloading (Stage 4).
2. Case 2: From final backfill level (Stage 3) to the end of preloading (Stage 5).
3. Case 3: Total settlement from the start of backfilling (Stage 1) to final consolidation (Stage 5).
4. Case 4: From the addition of the final structure load (Stage 7) to the end of consolidation settlement (Stage 8).

The settlement values were analyzed for different CoV values, and results are summarized in Figures 6–9.

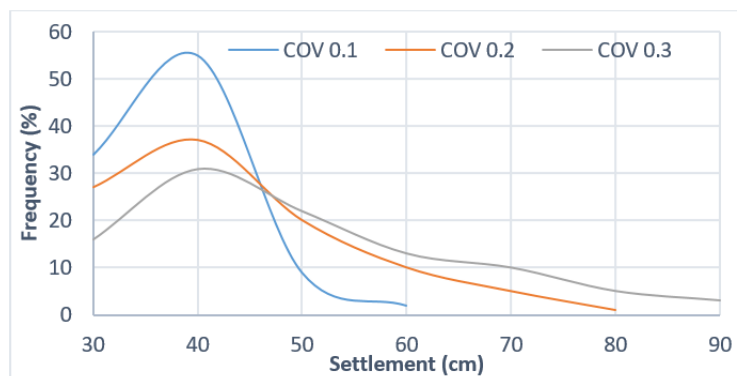


Fig. 6. Reliability analysis results for different CoV - Settlement case 1 (Interval 10cm)

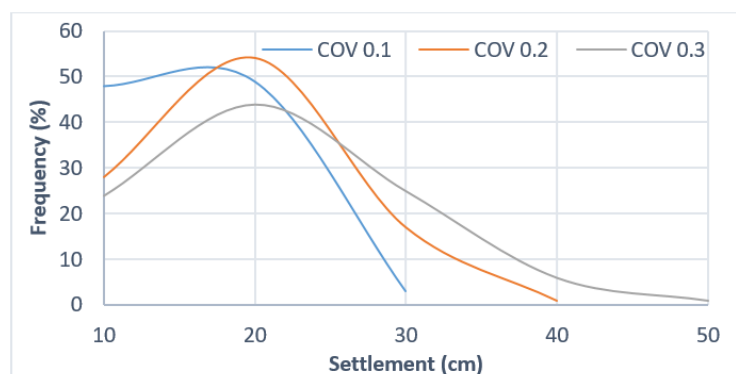


Fig. 7. Reliability analysis results for different CoV - Settlement case 2 (Interval 10cm)

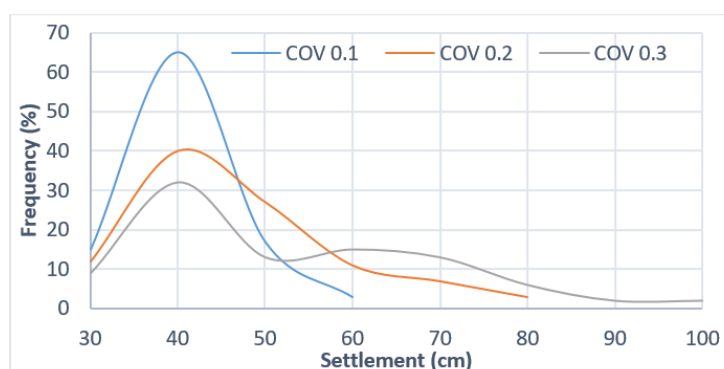


Fig. 8. Reliability analysis results for different CoV - Settlement case 3 (Interval 10cm)

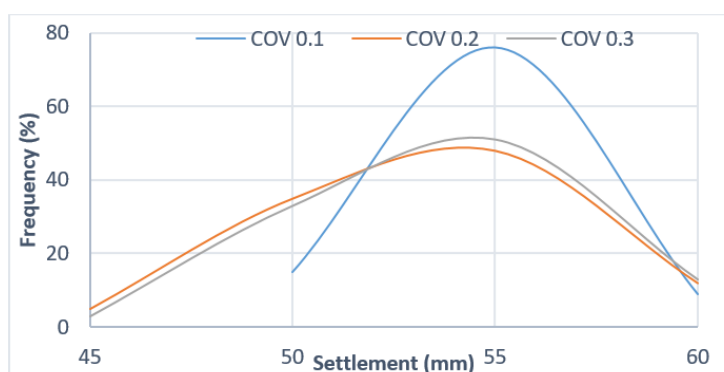


Fig. 9. Reliability analysis results for different CoV - Settlement case 4 (Interval 10cm)

The results of the previous figures can be summarized as follows:

- Figure 6 (Case 1):
 - Peak Settlement: 40 cm.
 - Frequency of Peak Settlement: 55% (CoV 0.1), 37% (CoV 0.2), 31% (CoV 0.3).
 - Settlement Range: 60 cm (CoV 0.1), 80 cm (CoV 0.2), 90 cm (CoV 0.3).
- Figure 7 (Case 2):
 - Peak Settlement: 20 cm.
 - Frequency of Peak Settlement: 49% (CoV 0.1), 54% (CoV 0.2), 44% (CoV 0.3).
 - Settlement Range: 30 cm (CoV 0.1), 40 cm (CoV 0.2), 50 cm (CoV 0.3).
- Figure 8 (Case 3):
 - Peak Settlement: 40 cm.
 - Frequency of Peak Settlement: 65% (CoV 0.1), 40% (CoV 0.2), 32% (CoV 0.3).

- Settlement Range: 60 cm (CoV 0.1), 80 cm (CoV 0.2), 100 cm (CoV 0.3).
- Figure 9 (Case 4):
 - Peak Settlement: 55 mm.
 - Frequency of Peak Settlement: 76% (CoV 0.1), 48% (CoV 0.2), 50% (CoV 0.3).
 - Settlement Range: 45 mm to 50 mm.

Based on the above results, the following can be observed:

- As CoV increases, the range of settlement values widens, indicating greater uncertainty in predictions.
- Frequency of peak settlement decreases with higher CoV, reflecting the influence of variability in input parameters.
- Settlement values and variability trends highlight the importance of accurately estimating CoV to predict settlement behavior effectively.

8.3 Structure Stresses

To evaluate the impact of structure stresses on settlement reliability, stresses of 20 kPa, 30 kPa, and 40 kPa were analyzed using 100 iterations. The analysis focused on Case 4 (effective settlement of the structure), with medium certainty (CoV = 0.2) for consolidation parameters of the seven upper clay sublayers. Results are presented in Figure 10.

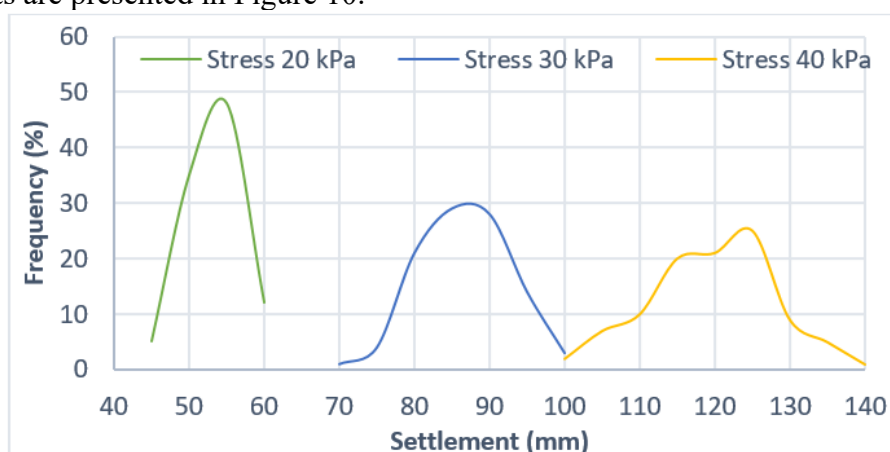


Fig. 10. Reliability analysis for structures effective settlement results (Interval 5 mm)

The results of the previous figure (Figure 10) can be summarized as follows:

- Peak Settlement Values:
 - 20 kPa: 55 mm.
 - 30 kPa: 85 mm.
 - 40 kPa: 125 mm.
- Frequency of Peak Settlement:
 - 20 kPa: 48%.
 - 30 kPa: 29%.
 - 40 kPa: 25%.
- Settlement Range:
 - 20 kPa: ~15 mm.
 - 30 kPa: ~30 mm.
 - 40 kPa: ~40 mm.

Based on the above results, the following can be observed:

- Higher structure stresses result in greater settlement values and wider settlement ranges.
- Frequency of peak settlement decreases as stress increases, demonstrating the influence of applied stress on the variability of settlement.
- Accurate prediction of stress-induced settlement requires careful consideration of the stress levels and associated variability.

9 Conclusions

This research investigates the uncertainties in soil consolidation parameters and their impact on predicting settlement in soft clay soils treated with a combination of preloading and prefabricated vertical drains (PVDs). The study uses the PLAXIS 2D finite element software, integrated with Python scripting, to perform stochastic analyses through Monte Carlo simulations. The focus is on assessing the influence of soil compressibility parameter variability on settlement reliability, examining three key factors:

1. The relationship between the number of Monte Carlo simulations and the precision of settlement outcomes.
2. The effect of the coefficient of variance (CoV) of soil consolidation properties on the results.
3. The influence of structural stresses on settlement reliability calculations.

The research conclusions can be summarized as follows:

1. Monte Carlo simulations are shown to be effective for settlement reliability studies.
2. The integration of Python scripting with PLAXIS provides a robust framework for analyzing settlement reliability, particularly in cases involving PVDs and preloading systems.
3. Accurate characterization of soil variability is essential for reliable settlement predictions.
4. Estimating statistical parameters such as the mean (μ), standard deviation (σ), and coefficient of variance (CoV) is critical for comprehensive reliability assessments.
5. The precision of settlement predictions improves with the number of Monte Carlo iterations:
 - 1000 iterations offer the highest precision.
 - For preliminary designs, 100 iterations can be used, though the error margin may reach 27%.
 - Error percentages for 200, 400, 600, and 800 iterations are 16.2%, 11.6%, 8.3%, and 5%, respectively.
6. The number of iterations should be chosen based on project significance and the sensitivity of the structure.
7. Higher CoV values, reflecting greater uncertainty in consolidation parameters, result in:
 - Reduced confidence in settlement predictions, with frequencies potentially decreasing by 50% compared to low CoV scenarios.
 - A significant increase in the range of settlement values, potentially doubling the expected variability.
 - Higher maximum settlements, which could critically impact structures sensitive to deformation.
8. Increased structural loads lead to:
 - Lower certainty in predicted settlements.

- Wider settlement ranges.
 - Higher maximum settlement values, even if the probability of occurrence is low.
9. The degree of uncertainty in soil parameters strongly influences settlement outcomes.
 10. Enhanced site investigations and careful consideration of variability can significantly improve settlement reliability.
 11. The findings highlight the importance of balancing uncertainty factors and design precision, especially for critical infrastructure projects.

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